CHAPTER 1: INTRODUCTION

One of the most difficult topics to teach in geometry is the concept of proof. In one national study, less than a third of geometry students gained a mastery of proof in their geometry classes, and typical high school mathematics programs provided virtually no opportunity for students to continue developing proficiency in formal proof after the geometry course (Senk, 1985). Yet still proof as a concept and skill is highly valued in the mathematics community and remains a requirement in both state and national mathematics standards across the country.

Project Overview and Research Question

In this study, I examined the impact of using a highly visual approach called ProofBlocks in teaching proofs in six high school geometry classes. I then compared results from these classes with results from five classes in which proofs were introduced initially and taught primarily in two-column proof format. The research took place over an eight week period at two different schools with vastly different student populations. Data collected included a written exam on proof of triangle congruence, and two sets of attitudinal surveys: one administered before instruction in proof and the other upon completion of the unit on congruent triangles. Results from these sources were triangulated to answer the following research questions:

1. *Achievement*: What impact does teaching ProofBlocks have on students' development of basic competence in proof as measured using a common written assessment?

2. *Confidence*: What impact does teaching with ProofBlocks have on students' self-reported confidence in their proof abilities as measured using student surveys?

Participating students took both the surveys and the common written assessment. I compared attitudinal survey data and student achievement data obtained within each school. Students' responses to the units were then analyzed in light of the two areas of interest stated above.

Rationale for Project

According to the National Council of Teachers of Mathematics (NCTM), the study of proof is so important that it should be incorporated at all levels of instruction. The organization calls for instructional programs from prekindergarten through twelfth grade to provide all students with the opportunities and skills to "develop and evaluate mathematical arguments and proofs" (NCTM, 2000, p. 341) as well as "select and use various types of reasoning and methods of proof" (p. 347). Aside from its primary importance, proof also is an activity which develops and fosters the communication skills which are one key focus for the NCTM *Principles and Standards*. In doing proofs, students organize their thoughts into logical patterns of reasoning and use precise mathematical language to communicate their discoveries, strategies, and conclusions to others.

Specifically with regard to the study of proof within a high school geometry class, *Principles and Standards* (NCTM, 2000) states the following:

> Judging, constructing, and communicating mathematically appropriate arguments… [remains] central to the study of geometry. Students should

> see the power of deductive proof in establishing the validity of general results from given conditions. The focus should be on producing logical arguments and presenting them effectively with careful explanation of the reasoning, rather than on the form of proof used…. (p. 309)

While this study focused on the impact of ProofBlocks on the skills and attitudes of students within a number of geometry classes, the emphasis of the instruction within these classrooms remained on scaffolding the development and presentation of logical geometric arguments. The ProofBlock format functioned as a tool to this end and not an end in itself. Proof in these classes was introduced using ProofBlocks, however, when students reached sufficient mastery of the patterns of rigorous argument, they were encouraged to use whichever form of proof would most efficiently and coherently convey their ideas.

Defining ProofBlocks

While nearly all proofs are presented in paragraph form at the university level, high school geometry texts have traditionally focused instruction on proofs written in two distinct columns. In these "two-column proofs," geometric or algebraic statements appear in the left-hand column and the corresponding theorems, definitions, or postulates supporting each statement appear in the right-hand column. Two-column proofs are generally more concise and easier to discuss and grade, than paragraph proofs written by students just learning the material.

In the early 1960's, evidence of a new form of proof began appearing in professional publications like *Mathematics Teacher*, and by the late 1970s, it was being referred to as a “flow proof” (Ness, 1962; Thorsen, 1963; McMurray, 1978). Flow proofs are, in substance, identical to two-column proofs. The main difference is that the statements, instead of being listed in columns, are connected one to the next with arrows showing the flow of reasoning. The theorems, definitions, and postulates generally appear either beneath the corresponding statement or on an arrow, presenting the reasoning required to connect two statements. This method is more visual than that of the two-column proof and more clearly connects related pieces of information. Most geometry texts now include (at the very least) examples of this form of proof as an option for teachers and students. In my experience, I have found that students usually like the visual nature of this format, but often still have just as much difficulty creating rigorous proofs with it as they do with two-column proofs. Many students unknowingly skip steps because though the flow of reasoning is obvious, they are still uncomfortable with the rules available to them to justify their individual steps.

Looking to meet the needs of the diverse learners in my own classroom three years ago, I developed ProofBlocks as an alternative to two-column proofs or flow proofs. Though the ideas behind them are simple, I designed ProofBlocks to provide an approach that is both intuitive and rigorous, a full understanding of which will allow students not only to solve geometry proofs, but will give them tools for crafting logical arguments and improving their general problem solving abilities. The format is inherently visual and easily lends itself to the use of manipulatives, having been designed in part to address the needs of the visual and kinesthetic learners who, in my experience, are often neglected in other forms of proof. The remainder of this chapter will discuss the details of both the ProofBlocks manipulatives and proof format, how students can design and connect blocks to form proofs, and how ProofBlocks can be applied to help scaffold the development of mathematical language.

The overall concept behind the ProofBlocks manipulatives is simple. Each theorem, postulate, and definition is represented by a unique two-dimensional manipulative with inputs that clearly indicate the conditions required to use it and outputs that list the conclusions it will allow you to draw. These manipulatives are called ProofBlocks, a few examples of which appear in Figure 1.

Inputs and outputs of different blocks can be connected when the conclusion of one exactly matches the requirement for the next. For instance, if students know that two angles in one triangle are congruent, they will be able to apply that information using the "Converse of Base Angles Theorem" block (see Figure 2). Depending on the geometry of the problem and what they are trying to prove, they might choose to use the "definition of congruent segments" block to convert the congruent segments to equal lengths for the purpose of algebraic manipulation. Alternatively, the congruent segments might also be used as one pair of corresponding sides required by the SSS Postulate block to prove two triangles congruent. Matching outputs and inputs of blocks in this manner is intended to lead naturally to the students construction of only

Figure 1 – Two dimensional manipulatives representing the definitions, postulates, and theorems students have encountered, help students apply their geometric understanding to proofs while scaffolding their development of logical reasoning skills.

logical arguments and to give them confidence, upon completion, that their proof is rigorous and correct. A more detailed description of how ProofBlocks work and the design of individual blocks is included in Appendix A.

"In order to make sense of mathematics and to communicate mathematical ideas, it is essential to be able to assess and produce mathematical arguments, including formal proofs" (Martin, et al. 2005, p. 95). This study explored a new method to address the difficulties students face when learning to prove geometric theorems, thus it was important to explore the recent literature related to the teaching of proof, the learning of proof, and the nature of what constitutes proof in the eyes of students, teachers, and mathematicians. What follows is a brief overview of the volumes of work on these topics.

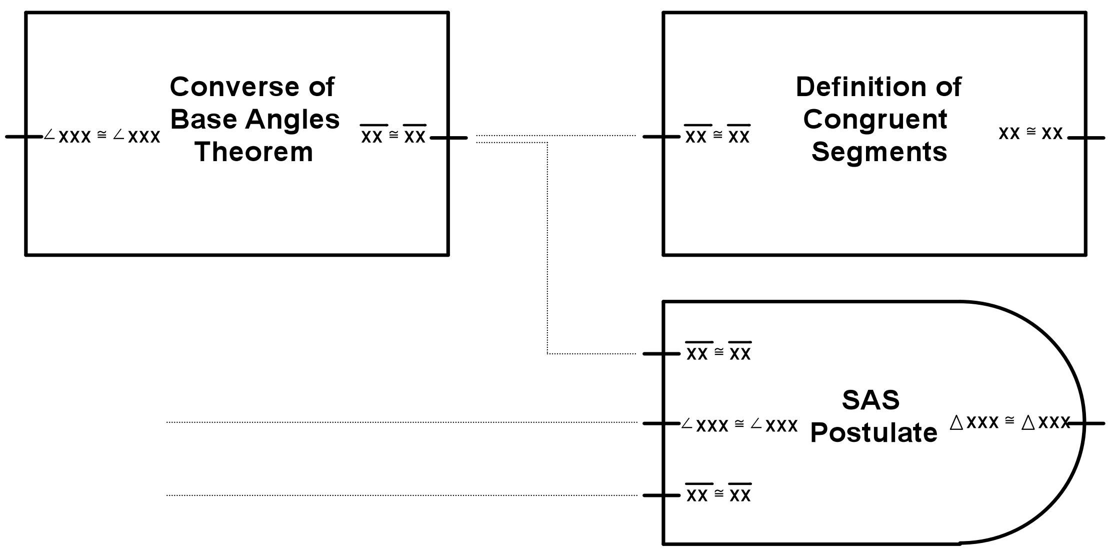


Figure 2 – Two blocks can be connected when the label on an output of the first block exactly matches the input of the following block. Like the theorems they represent, blocks may require more than one input or may generate more than one conclusion. Thus, the output of one block may or may not fully satisfy all the requirements of the following blocks and any missing information would have to come from other blocks.

CHAPTER 2: LITERATURE REVIEW

While NCTM calls for all geometry students to construct and communicate mathematical arguments, educators have been struggling for decades with how to reach such a goal. In a study conducted in the early 1980's, the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project concluded that almost a third of high school geometry students in the United States ever reached even a 75% mastery of proof (Senk, 1985). A number of studies around the world have reported finding significant gaps in the understanding of proof, not only at the high school level, but also with college students and preservice teachers (Martin & Harel, 1989; Chazan, 1993; Healy & Hoyles, 2000; Harel & Sowder, 1998).

There has been an increase in research in the last two decades into how students learn to prove geometric conjectures. The research findings in this chapter will be organized into three main topics: (a) a definition of proof and its role in the classroom, (b) factors contributing to students' poor performance on proof, and (c) the development of problem solving and reasoning skills.

Defining Proof and its Role in the Classroom

Traditionally, many teachers and textbooks focus solely on proof as a means of verification. While acknowledging that proof has other important roles, Hanna (2002) still states that "every student just entering the world of mathematics [must] start with the fundamental functions: verification and explanation" (p. 8). From his focused interviews of geometry students, Chazan (1993) concluded that "the explanatory aspect of proofs is a useful starting point for a discussion of the value of proofs" (p. 383). However, DeVilliers (1990) points to proof's other roles arguing that students are entitled to a more full understanding of the meaning and usefulness of proof, and when provided with such, may perform more competently in this very important piece of the high school curriculum.

Even for mathematicians, proof is seldom about step-by step verification of a result. Unlike the proofs traditionally posed by geometry texts, in which students are given the final result at the outset, mathematicians frequently reason deductively toward an unknown conclusion. Through this process of proof, mathematicians over the years have happened on a number of exciting and unexpected discoveries which have subsequently become the starting points for further mathematical inquiry (de Villiers, 1990). Mathematicians use proof to organize their results and thought processes in ways that establish a common basis for understanding, application, and critical debate. De Villiers highlights five key roles for proof relevant to today's research mathematicians: verification, explanation, systematization, discovery, and communication. Other researchers have added to this list. If these are the functions proof serves for mathematicians, he argues, our students as well should have access to proof in similar ways.

These suggestions, while met with excitement in many circles, are also problematic for many researchers. Balacheff (2002) notes that among educational researchers, there seems to be no coherent and accepted working definition for proof as it applies to students in a classroom, that beyond its formal definition, the nature of proof is a culture dependent entity. To address this confusion, Stylianides (2007) recently proposed the following definition of proof:

> Proof is a mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics:
>
> 1. It uses statements accepted by the classroom community (*set of accepted statements*) that are true and available without further justification;
> 2. It employs forms of reasoning (*modes of argumentation*) that are valid and known to, or within the conceptual reach of, the classroom community; and
> 3. It is communicated with forms of expression (*modes of argument representation*) that are appropriate and known to, or within the conceptual reach of, the classroom community. (p. 291)

One of Stylianides' key concerns is the need for clarification about the role of proof in elementary grades. This definition highlights the importance of critical discourse at an appropriate level: a key step in further developing proof as an accessible mode of communication as called for by NCTM. While it does not restrict the role of proof in any way, as described by de Villiers, it prevents empirical arguments from being considered a form of proof.

Much of the current research is focused on students' view of proof as empirically driven. Chazan (1993) notes that "as beginning provers, students do not have strong reasons to believe that deductive proofs guarantee safety from counterexamples" (p. 383), and cites a study by Fischbein (1982) in which only 24.5% of students polled who believed a proof to be correct also simultaneously acknowledged that that meant they did not need to continue searching for counterexamples. Similar results were obtained by

Reiss, Klieme, and Heinze (2001) in a study of geometry students in Germany. From studies of preservice teachers, Martin and Harel (1989) concluded that students develop an inductive frame for judging the validity of arguments sooner than they develop a similar deductive frame. Additionally, they found that this inductive frame is not immediately discarded in favor of the deductive frame. Thus, when approaching a proof, both inductive and deductive sets of arguments must exist for many students to believe a specific result. In her work with pre-algebra students, Ellis (2007) suggests that students' movement from empirical and non-deductive "proofs" to correct generalizations and rigorous proof occurs through a spiraling process of justifying and generalizing. This process is tied closely to the development of more complete concept images and more sophisticated proof schemes, both of which are discussed later in this chapter.

Students' impressions concerning acceptable formats of proof also affect their correct identification of sound deductive arguments according to Healy and Hoyles (2000), whose research focused on British students' views algebraic proof. Their results are similar to those of Chazan, Fischbein, and Reiss in that they found that empirical argument, or argument by example, was common even among students who recognized its limitations. Additionally, however, they observed that "students simultaneously held two different conceptions of proof: those about arguments they considered would receive the best mark and those about arguments they would adopt for themselves." Students in their study were generally more successful with a narrative style proof than with more formal proof, however, these same students also expressed the feeling that their teachers would give higher grades to the more formal proof. Along similar lines, Martin and Harel (1989) noted that of the elementary preservice teachers they studied, many were influenced by the appearance of the argument. While most correctly identified the correct deductive argument on their test as a proof with high validity, there were also a large number of false-positive identifications of the faulty argument presented in typical paragraph proof format. In Knuth's (2002a) research, more than one third of non-proofs shown to teachers were rated as proofs. In light of these findings, it seems worthwhile that teachers consider more fully the format of proof taught and accepted in the classroom and the impact this format has on students' understanding of what constitutes a valid proof.

Factors Contributing to Student Performance on Proof

In the mid-1980's, Dina van Hiele-Geldof and Pierre van Hiele presented a model for the development of geometric thought. While it has been amended and advanced in the ensuing years, it has come to be used as a tool for assessing student understanding of geometric concepts and by some as a means of guiding instruction (Crowley, 1987). It also lends insight into why students have a hard time with proof in traditional geometry courses. According to the van Hieles, students progress sequentially through different levels of geometric thought. This progression is less dependent on the age of the learner than on the curriculum and methods of instruction employed. There is some disagreement in the literature as to exactly how to number the levels of understanding in this model, but for the purposes of this paper, I will use the following number scheme: (1) visualization, (2) analysis, (3) informal deduction, (4) deduction, (5) rigor.

At the visualization level, according to Senk (1989), students "recognize common geometric figures but [are] unable to describe properties of those figures" (p. 318). For instance, a student operating at level 1 would be able to identify shapes such as squares and rectangles, but would not recognize that they both have right angles and that a square is a special case of a rectangle. At level 2, students can classify shapes by their properties and make generalizations about these classes. However, they still do not form chains of deductive reasoning until they reach the third level, informal deduction, at which point they begin to be able to follow formal proofs and generate informal arguments themselves. The van Hieles claimed that students at this level were incapable of generating their own proofs, but Senk's research has not supported this claim. Regardless, students at this level have generally been found to combine empirical and deductive arguments in their reasoning, feeling uncomfortable relying entirely on deduction. At the fourth stage of geometric thought, known as deduction, students can construct formal proofs on their own. When students reach the final stage, geometry has become abstract for them and they can work effectively in both Euclidean and non-Euclidean geometries.

Using the data from part of the CDASSG project, Senk (1989) found that the van Hiele level at which a student enters can be a useful predictor of achievement in geometry. Students beginning a high school geometry course unable to identify and name simple shapes and figures (operating below level 1 in the model), are very unlikely to learn to write proofs by the end of the course. Students entering the course at level 1 will probably be able to do simple proofs, but have little chance of mastery (less than a 33% chance). Entering the class at level 2, able to identify, classify, and describe properties of shapes, students have a 50% chance of mastering proofs by the end of the year. For students entering the course at the higher levels of the model, Senk found that van Hiele level became less effective as a predictor of students' performance. Geometric content knowledge (as distinguished from sophistication of geometric thought) played a greater role in determining student success at these levels than it had at the lower levels.

Given the stratification of scores described above, it is worth noting that proof writing achievement has not been found to divide students into two distinct groups: those who can write proofs and those who cannot. On the contrary, in a 1985 article in *Mathematics Teacher*, Senk reported finding that "25 percent of the students [in her study] have virtually no competence in writing proofs; another 25 percent can do only trivial proofs; about 20 percent can do some proofs of greater complexity; and only 30 percent master proofs similar to the theorems and exercises in standard textbooks" (p. 453-454). In addition, some individual students who tested at the lowest van Hiele levels did indeed manage to master proofs by the end of the year.

The levels are not perfect predictors, but they do serve as a useful tool in determining the form of instruction appropriate to each student. "If the student is at one level and instruction is at a different level," Crowley (1987) asserts, " the desired learning and progress may not occur... [because] the student will not be able to follow the thought process being used" (p. 4). She goes on to state that most geometry courses are taught at level 4: deduction. However, it is clear from the studies cited above that most students are not operating at this level, especially early in the year. It appears likely that the discrepancy between the level of instruction or curricular materials and the level of the geometric thought that many of students demonstrate may be partially responsible for many students' failure to master proof.

Another factor posited to have contributed substantially to students' difficulties learning proof is the abrupt way in which students are introduced to it (Stylianides, 2007; Moore, 1994; Ball, 2002). Though the NCTM Standards call for proof activities in all levels of school mathematics, these activities have traditionally been limited to high school geometry classes. There has been very little formal or informal proof up to that point, and according to Senk (1985), little follows until students reach college. She observes that this "mastery of many mathematical skills is not accomplished at the time of their major emphasis in the curriculum" and this mastery "occurs only after a skill has been used for a length of time and in a variety of contexts" (p. 454). Thus, this lack of emphasis outside a single year at least partially explains the low level of comfort with formal proof described by Moore (1994) and the Seldens (1995).

Both Senk (1985) and Moore (1994) identified some key issues with which students struggled when approaching proof problems. Though Moore's research was performed with a group of calculus-level college students and Senk's was with high school students, both researchers observed that students had noticeable difficulties starting proofs. Moore also points to mathematical language and concept understanding as issues to be addressed if students are to find more success. Senk suggested that teachers should focus more on the meaning of proof, as a number of students tried to use the theorem to be proven as part of their proof. She also observed that many students are unsure of when and how to transform a diagram (adding auxiliary lines or searching for overlapping or embedded figures) to make it possible to complete a proof. Finally, Moore noted that "the students appeared to be overwhelmed by the necessity of grappling with difficult group theory concepts, problem solving, abstraction, and generalization while learning what a proof is and how to write one" (p. 265). While this observation was made of university students, the same could likely be said of high school geometry students.

Of possibly greater concern are the findings suggesting that many preservice, elementary, and experienced secondary teachers have only a limited understanding of what constitutes proof (Martin & Harel, 1989; Knuth, 2002a; Stylianides, 2007). Much of this research focuses on the misidentification of inductive arguments as a valid type of proof. In Martin and Harrel's (1989) study, over eighty percent of preservice elementary teachers identified an inductive argument as a mathematical proof. Teachers who saw proof as formal, rigorous argument in Knuth's (2002b) research did not feel that it was an appropriate and accessible topic for all students. If proof is to be taught effectively to all students, teachers must reexamine their own conceptions of proof.

Development of Problem Solving and Reasoning Skills

As stated earlier, Moore (1994) observed that many university students were overwhelmed by the sheer number and difficulty of mathematical and cognitive tasks they faced while attempting to learn to create a proof. How much more so must this be the case when students are first exposed to the concept of proof? Through his research in higher education, Moore (1994) has developed a model of major sources of difficulties students run across in proving, shown here in Figure 3. Note that this model is not meant to illustrate *how* students think about proof, but is more a method of visualizing the interconnectedness of a variety of student perceptions, misconceptions, and gaps in understanding when it comes to students' struggles with proof.

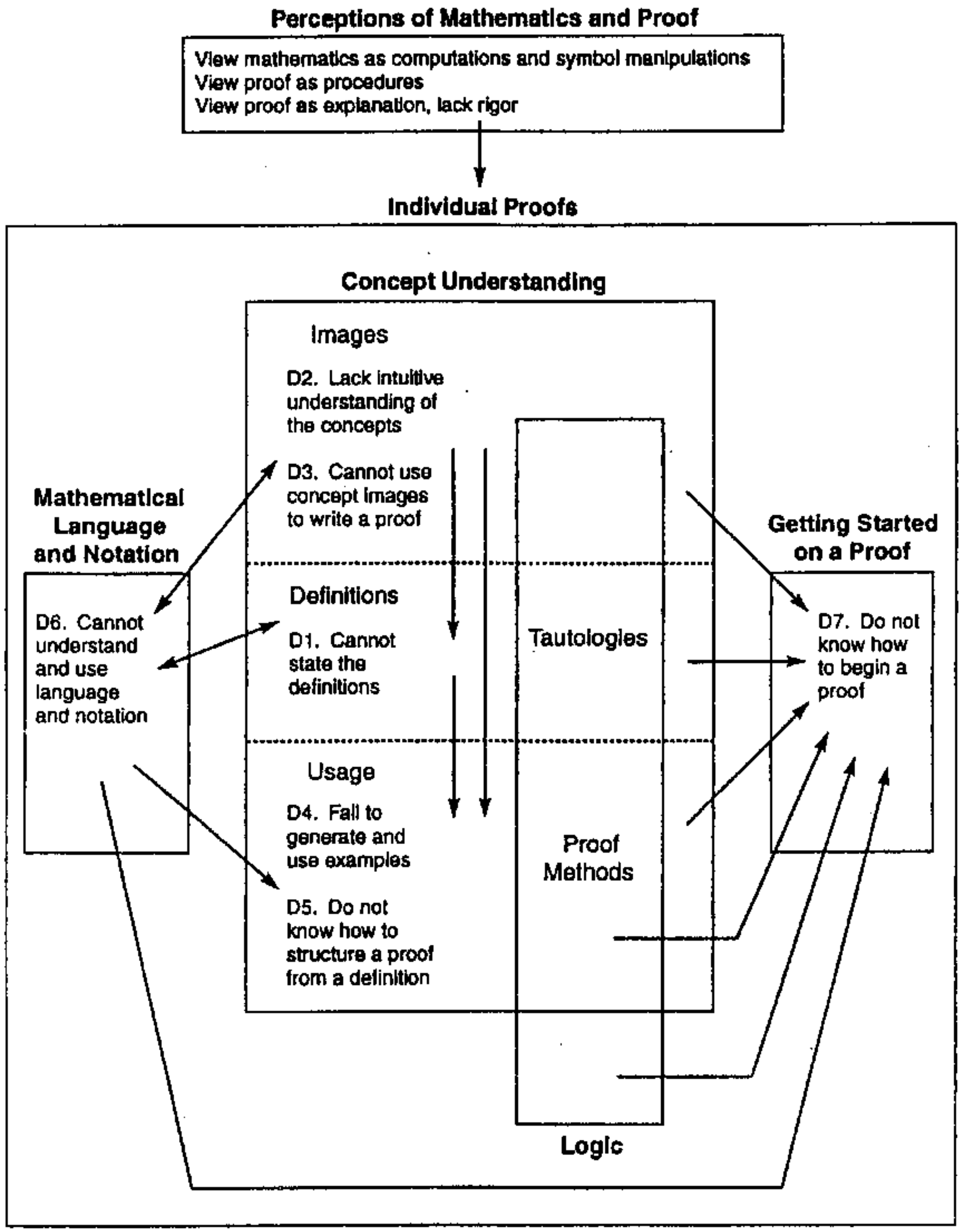


Figure 3 – Model of the major sources of students' difficulties in doing proofs (Moore 1994, p.253)

In this model, Moore begins by recognizing the key role which students' perceptions of proof play in their ability to successfully master proving. Students who view proof solely as an informal explanation are unlikely to generate mathematically rigorous proofs. So too will students with inadequate concept understanding or grasp of mathematical language fail to meet acceptable standards for proof. Taken all together, it is clear that there may be many reasons for students to experience difficulties beginning a formal proof.

It may become clearer where students are likely to stumble when learning to prove if, rather than looking at the prerequisites for proof as a web of interconnected skills, each step is analyzed individually. In 1945, Polya presented a general problem solving approach which, when applied to proof, helps to break down the process into distinct definable segments. The method has four steps: first, to understand the problem; second, to find the connections between the data given and the unknown, and then using these connections to generate a plan; third, to carry out the plan; and finally, to examine the solution.

Polya places great importance on the first step of understanding the problem and identifying the goal. Unfortunately, for many students, this step is the one restricting their access to the rest of the proof process. Laborde (1990) notes that "language is needed to introduce students to new notions and that language may turn out to be an obstacle to students understanding" (p. 69). Those students who have inadequate understanding of the mathematical language and vocabulary have difficulty beginning their own proofs, but also must inevitably struggle to understand the proofs of other students in order to effectively learn from them. Moore (1994) identifies a further deficiency in many students' concept images which also explains their inability to grasp the full meaning and implications of the problem.

These concept images, as described by Moore, are essentially mental pictures which students form of the mathematical concepts: informal images that help students to frame the meaning of a problem. Having concept images which are not sufficiently advanced to cope with a problem statement, leaves students unable to understand the context, direction, or intent of the problem, thus crippling their ability to approach it effectively. For instance, imagine a student whose concept image of a trapezoid is an isosceles trapezoid. Informally, she can describe the shape and distinguish it from other quadrilaterals. However, when asked prove that the length of one leg is longer than the length of the other, she will run into a contradiction between her concept image of a trapezoid, and the formal definition which does not depend on the legs being equal in length. Until this conflict is resolved, the proof itself holds no meaning for her and she is likely to find herself incapable of beginning the process.

The second step in Polya's method involves connecting the given information with the anticipated final result, or at least planning a process that will lead from one toward the other. According to an article in *Cognitive Science* by Koedinger and Anderson (1990), experts in geometric proof focus on key steps and skip the less important ones. The authors suggest that a starting point for experts is to identify familiar configurations in the given diagram. To these familiar configurations (for instance, parallel lines cut by a transversal or vertical angles) are connected a series of related theorems, definitions, and postulates, which can be sorted and evaluated based on their relevance to the given information. For students operating at the lowest van Hiele levels, who have not yet learned to analyze diagrams with regard to their properties, this process is next to impossible. In addition, for years teachers have been pointing to students' struggle to memorize or remember theorems as a hurdle to successful proof completion (Ness, 1962). However, the issue may be slightly more complicated than just a lack of memorization. Speaking of a professor whose class he was observing, Moore (1994) noted that "the superior organization of his knowledge structures reduced the cognitive load whereas the students often suffered from cognitive overload" (p. 262). Regardless of the reason, without the ability to quickly recall relevant theorems, students have little opportunity to plan proofs in the way described above.

Many times, Moore (1994) notes, students could also plan a proof by taking structural cues from formal definitions. For instance, given a theorem in conditional ("If-then") form, students often do not recognize the hypothesis and the conclusion of the conditional as given statement and a statement to be proven. The definition, in fact, suggests a sequence of steps, but students are not comfortable enough with the formal definitions to find this useful in planning (p. 258). Other researchers agree that difficulties identifying the logical structure of a statement will negatively impact students' ability to construct a logical proof structure for the statement (Selden & Selden, 1995).

Following the planning phase of Polya's method, students are expected to carry out their plan, following a chain of logical reasoning. Heinze and Reiss (2003) suggest that this aspect of students' understanding is difficult to test or measure because it is interconnected with so many other aspects of the proof process. They suggest that students have less trouble with this aspect of proof so long as the material with which they are working is familiar.

The final phase of Polya's approach to problem solving is to examine the solution that has been reached. At this point comes the twofold difficulty of checking for validity and determining whether or not the results are generalizable. To check for validity, students must determine whether or not each step of the proof follows from earlier steps and a valid original assumption. Then they must decide whether or not the statement proven was indeed what was meant to be proven. The Seldens (1995) noted that some students, even at the university level, have weak validation skills, and recommended that these be improved through an increased emphasis on defense of their solutions to mathematical problems outside the realm of proof as well. However, while developing a less formal set of validations skills may be an important first step, checking for suitable levels of rigor and continuity of argument will likely still remain difficult for these students. The Seldens found that many of the students in their study saw explanatory comments or tangential discussion as part of a proof, instead of supplemental to it. Moore (1994) suggests that students will not be able to see the importance of a precise, formal definition if their understanding of the concept goes no deeper than an informal explanation.

It has already been discussed previously how some students, when learning to create proofs have little confidence in a purely deductive solution. An important part, then, of convincing themselves early in the process that their result is correct, is for students to generate illustrative examples and search for counterexamples. Moore (1994) observed that students participating in his research had a tendency to generate examples which were more complicated than they could manage to analyze. The mathematical activity that the professor engaged in when approaching and analyzing the problems, namely generating simple relevant examples that elucidated key aspects of the problem or solution, was out of reach for these students. One approach adopted by some teachers to help students generalize and create their own examples, has been to refocus the curriculum on forming conjectures and the search for counterexamples. Martin, McCrone, Bower, and Dindyal (2005) recommend from their research the use of conjectures and open-ended tasks to encourage increased discourse. Evaluating student produced arguments in this more informal context, students may develop more comfort generating relevant examples while not confusing these examples with the proof itself (Selden & Selden, 1995).

Senk and Moore both agreed that one reason many students fail at proofs has to do with deficiencies in concept understanding. In addition to increasing mathematical discourse in the classroom and bringing the concept of proof to the elementary grades, a number of other research-based methods have also been proposed to address this difficulty. Crowley (1987) recommends a change in instruction and instructional materials to coincide with "phases of learning" as described by the van Hieles. These phases involve the use of discourse and open-ended problems to scaffold students' transitions from one level to another. Hanna (2000) recommends the use of dynamic geometry software to help students develop more effective visualization skills and provide access to exploratory proofs.

Educators have also been examining and improving techniques to allow students to work with visual representations of proofs. For example, Ness (1962) and Thorsen

(1963) suggested the use of structured diagrams similar to flowcharts. These were followed by Hallerberg's (1971) schematic form and McMurray's (1978) flow proof. More recently, work in this field has focused less on altering format and more on the process of deepening geometric understanding through conjectures made prior to actual proof (Cox, 2004). Others have chosen to add context to make the logic and the diagrams more intuitively understandable, one example of which would be the use of Crane and Rubenstein's (2000) airline route metaphor.

"Much of a student's achievement in writing geometry proofs is due to factors within the direct control of the teacher and the curriculum" according to Senk (1989, p. 319). ProofBlocks is one approach to addressing the achievement dilemma with regard to proofs which is evident in today's geometry classrooms and which was described in the research literature. While it does not itself act as a tool for developing more complete concept images, it does provide a way of minimizing the cognitive overload many students experience when asked to memorize, summarize, or organize a large number of theorems in their heads. Providing students with an "encoded" version of each definition, postulate, or theorem in manipulative form, allows students to transfer what used to be a mental process to the desk in front of them, sorting through, evaluating, and choosing those tools which are applicable to the proof at hand. Not only does this process help students, but with the thought processes so clear for observation, teachers can more easily identify misconceptions and guide student work.

In the course of instruction, students transition away from their reliance on the manipulatives, while still retaining the very visual ProofBlock format. At this point, the cognitive demand of the proof itself has not been diminished: the requirement to create a precise, relevant, and logical chain of reasoning still remains. However, the initial use of the ProofBlock manipulatives helps to scaffold the development of this reasoning as students progress through the van Hiele levels of geometric thought, toward an abstract understanding of proof.

I designed the research project with both the current literature and my own classroom experiences in mind. The next chapter will discuss the context and methodology of this project in detail.

CHAPTER 3: METHODOLOGY

The objective in this study was to determine what effects, if any, teaching using ProofBlocks would have on student confidence and achievement on geometric proof. The students in the classes receiving instruction with ProofBlocks began with proof of congruent triangles where the other students began proofs with angle pairs and parallel lines. In designing the study, I decided to allow students in the traditional classes a longer period of time before assessment in order to provide all students the opportunity to demonstrate their proof ability directly following the unit on triangle congruence, as shown in Figure 4. At the end of both of these units, all students took written exams, examples of which are included in Appendix D.

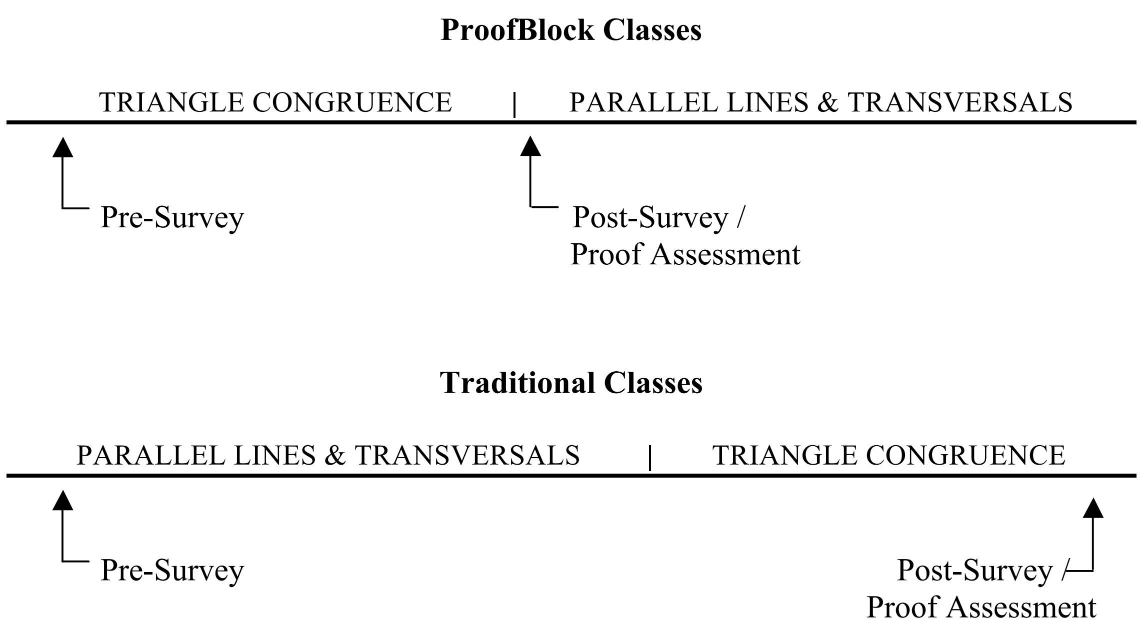


Figure 4 – Teachers instructing with ProofBlocks introduced proof in the unit on congruent triangles where the other teachers chose to begin with proofs involving parallel lines. In all classes, the pre-survey was given just prior to the introduction of proof while the post-survey and assessment followed triangle congruence. This design decision meant that students in the traditional classes had been working on proofs for approximately twice as long as students in the ProofBlock classes when they took the assessment and reported their confidence on the post-surveys.

This decision to assess performance on proof following the common unit instead of following a common time period for instruction was based on two considerations. First, using a shorter treatment period for ProofBlocks provided the possibility of highlighting the effectiveness of instruction in this format. Second, testing following a common unit allowed me to ignore issues surrounding retention of geometric knowledge and skills which might otherwise have been significant factors. All participating teachers were given the option to introduce geometric proof in whichever unit or through whichever method they believed most likely to be effective with their students. All those opting to teach via two-column proof also chose to follow the order prescribed by the student textbooks, introducing the concepts through proofs about parallel lines and related angles. The teachers using ProofBlocks elected to begin with proofs of congruent triangles and follow these with the unit on parallel lines.

In addition to using this quantitative measure, pre- and post-surveys were given to both groups of students as a means of gaining insight into the effects ProofBlocks might have had on students' attitudes toward proof. The pre-survey was designed to establish a baseline for understanding student views of proof upon entering the unit. Students were also asked to indicate prior experience with geometric proof, as a number of them were repeating geometry. The post-survey was administered following the unit on congruent triangles. The purpose of the post-survey was to allow me to observe the direction in which student responses might evolve as a result of the treatment. While both surveys contained a number of questions of interest to the researcher, for the purposes of this study only student responses to the question regarding their level of confidence with certain geometry problems was analyzed. A further description of the surveys is included in the section entitled Data Collection Methods.

Participants

Eleven high school geometry classes were used for this study; three classes from School A and eight from School B. The classes at School A were each taught by one of two teachers and composed entirely of eleventh and twelfth graders. These particular classes were chosen because they were typical of geometry classes taught to this age student at the school. Three of the four geometry teachers at School B were able to participate in the study, so data has been collected for all but one class of geometry students at the school. Each of the classes at School B contained students from grades nine through twelve. The site was chosen because it contrasts sharply with School A (see Table 1). The two schools are distinctly different with regard to size, racial and ethnic makeup, socioeconomic status, and academic achievement as measured by state tests. Taken together, the data sets from the two school sites provide a fuller answer to the research question than could just one alone.

Description of School A

School A was opened in 1956 and is one of ten high schools in its district. It is located in an urban area and is designated a Title 1 school, meaning that more than 40% of its students come from low income families. It had a total population of 3,559 students enrolled for the 2006-2007 school year, approximately 85% of whom qualified for free or reduced lunch. The student body was predominately Hispanic (91%) with White (2.7%) and Filipino (2.6%) accounting for the two next largest populations. Approximately 40%

	School A	School B
Schools	▪ Urban ▪ 3,559 students ▪ 91% Hispanic ▪ ~ 40% classified as English Learners	▪ Suburban ▪ 1,288 students ▪ 74% White ▪ ~ 1% classified as Limited English Proficient
Schedules	▪ Eight week semesters ▪ 95 minute block periods	▪ Twenty week semesters ▪ Primarily 60 minute classes with 110 minute blocks once per week
Teachers	▪ 2 Teachers - 4th year teacher (ProofBlocks) - Teacher with more than 5 years (Traditional) ▪ 3 Classes - 2 ProofBlocks - 1 Traditional	▪ 3 Teachers - 1st year teacher (ProofBlocks) - 2nd year teacher (ProofBlocks) - 4th year teacher (Traditional) ▪ 8 Classes - 4 ProofBlocks - 4 Traditional
Number of Student Participants	▪ 68 Students in ProofBlock classes ▪ 33 Students in traditional classes	▪ 129 Students in ProofBlock classes ▪ 125 Students in traditional classes

Table 1 – Statistics regarding the schools, teachers, and classes participating in the study.

of the student body was designated English Learners (EL) compared with 33% in the district.

It operates on a three track calendar with eight week semesters. Students are enrolled in four classes at a time. Each class met every day and lasted 95 minutes on a regular day. While the school has an academic magnet program in addition to ninth and tenth grade cohorts, the classes chosen for this study were juniors and seniors enrolled in regular geometry classes.

Description of School B

School B is located in a predominantly white, upper-middle class suburb of Los Angeles. While the average socio-economic status of the surrounding population is relatively high, most families have two parents employed full time outside the home. The site for the school was converted from a middle school campus ten years ago, and since that time its enrollment had grown by over 400 students. At the time of the study, the total enrollment was approximately 1,300 students.

The school is a performing arts and technology magnet high school, offering expanded programs in these areas, but lacking any California Interscholastic Federation (CIF) athletic programs. Designated as a "school of choice," it has no attendance boundaries and draws students from two neighboring school districts. The school actively recruits students from local middle schools who are interested in the specialized programs. While interested families are required to submit applications for enrollment, there is no selection process and students are accepted in the order in which their applications are received.

In the year of this study, the student body was predominately white (74%) with Hispanic (12.88%) and Asian (5.32%) accounting for the two next largest populations. Only 1.2% of the student body was designated Limited English Proficient (LEP) with another 8.84% designated Fluent English Proficient (FEP). In the year prior to the study, 11% of the student population qualified for free and reduced lunch.

The Treatment

Teachers maintained a great deal of autonomy as the study sought to minimize impact on normal teacher-student interactions, activities, and assessment. All teachers except the first year teacher at School B had taught in prior years with the proof method they used in this study. The first year teacher was given a choice between the two methods and decided on ProofBlocks in large part because of the additional planning resources she was offered to support this method.

In addition to a set of manipulatives, teachers electing to use ProofBlocks received a binder of resource materials which included worksheets, activities, and lesson plans which I had developed and taught my own classes from in previous years. The textbooks used at each high school expected students to be introduced to proof in the unit on angle pairs and parallel lines. Thus, many of the practice problems congruent triangles chapter required students to use properties of parallel lines in their proofs. ProofBlocks were designed to scaffold the learning of proofs through triangle congruence, and at this time have not been extended for use in other contexts. In anticipation of the more algebraic proofs of angle relationships in the upcoming chapter, all students were expected to transition into either two-column proof or paragraph proof by the end of the unit on triangle congruence. To provide more appropriate problem sets given the differences in the sequencing of material to that point, students who were taught with ProofBlocks were provided daily with supplemental worksheets, the problems on which required no background in parallel lines. Near the end of their unit on parallel lines, these students revisited triangle congruence, completing many of the textbook proofs requiring both sets of knowledge.

Though teachers in ProofBlock classes received planning resources, they were not required to use them. This study included no observations in part to allow the teachers to work with ProofBlocks in whatever way they deemed appropriate for their students. While they reported using many of the provided resources, at least one teacher was actively involved in redesigning and adapting the old lessons to meet the needs of this new group of students. I recommended that they provide students with the ProofBlock manipulatives for at least the first three days of proof instruction, which in my experience had been the length of time that it took for most students to stop asking for them, though some teachers find reason to leave them with the students for longer.

In addition to planning resources and manipulatives, teachers working with ProofBlocks also had access to a class set of large (two foot by 4 foot) whiteboards for student use at their desks. In all ProofBlock classes, each of these whiteboards was shared by a team of two or three students. All the ProofBlock teachers reported assigning students to show their work on these whiteboards for at least the first few days of their introduction to proof because they felt that the use of the whiteboards encouraged discussion and allowed students the freedom to take risks and make mistakes where they could be easily erased and corrected.

As there were no formal or informal observations made of any of the participating classes, and thus it cannot be assumed that the lessons in one method were necessarily more dynamic or the level of student engagement necessarily higher in one set of classes than in another. Nor can it be assumed that teachers adjusted their regular practice for the purposes of this study. All teachers were free to choose the style of instruction most comfortable for them and taught with the best interests of their students in mind.

Data Collection Methods

In this study, all students completed two attitudinal surveys (Appendices B and C) and one written exam (Appendix D). Given that the student populations at each site were so different and were being taught from different texts, I made the decision to allow a different exam to be given at each school. All students within the same school took the same exam, however. I wrote the exam given at School B and the ProofBlock teacher at School A wrote the test given to students at her school. Participating teachers were provided with these tests ahead of time so that they would be able to plan accordingly. The exam was given to both groups of students following the unit on congruent triangles. It allowed me to examine the level of student achievement in each group by assessing students' competency in proving triangle congruence. Each student's response was scored based on a common rubric which assigned points based on content and organization without regard to the form of proof selected (i.e. two-column, ProofBlock, or paragraph proof). The rubric may be found in Appendix E.

The questions on the exam were all free response and of varying degrees of difficulty, allowing the teachers to distinguish between several levels of performance. Though teachers scored each assessment in its entirety based on their own rubrics, only one proof on each assessment was used for the purposes of this study. This problem required students to use common definitions and geometric relationships to analyze a picture and then decide whether to use the SSS Postulate, the SAS Postulate, the ASA Postulate, or the AAS Theorem to prove the triangles congruent.

Within the category of achievement, this study sought primarily to examine the process of introducing and establishing a basic level of competence with proof, which could serve as a basis for building a deeper understanding through continued instruction throughout the rest of the course. While the problems on the exam were comparatively basic and did not make it possible to distinguish between students with advanced and moderately high levels of skill, this distinction was not within the scope of this study.

At this point it must be made clear that while students were allowed to draw out their proofs with shapes for theorems and geometric information labeling the connecting lines, no student was allowed to use the ProofBlocks manipulatives that they had relied upon at the beginning of the unit. All students in both classes were expected to be able to complete the proof without notes or assistance of any kind. Given these conditions, student responses were accepted in either the two-column or ProofBlock format, both being considered equally rigorous.

The pre-survey and post-survey were created as a means of gauging the effects of using ProofBlocks on students' attitudes toward proof. The pre-survey was administered within two days of the start of the first unit taught on geometric proof. The post-survey was given within a week after finishing the unit on congruent triangles. The pre-survey and post-survey consisted of seven Likert scale response items, some with multiple parts. The two surveys were very similar with regard to appearance, directions, and the substance of the questions. Only one question was included in this analysis. On both surveys, students were asked not to solve, but instead to rate their confidence on each of three problems on a scale of 1 to 4 where 1 indicated that they were "confident" about their ability to solve the problem and 4 indicated that they were "not confident." Each problem presented in the post-survey required identical skills to those in the pre-survey: one was a perimeter problem requiring students to sum expressions for the lengths of the sides of a quadrilateral and solve for *x*, one was a word problem requiring students to use and identify a pattern, and the final one was a proof of congruent triangles requiring the use of the reflexive property and/or one definition.

The post-survey, in addition to the Likert scale response questions described above, also included open ended questions concerning what students had found challenging in the geometry course in which they were enrolled and what their teacher had done that had made learning easier. The goal of these questions was to elicit comments that would provide insight into student learning challenges and how they had been addressed and, possibly, overcome.

For the purposes of this particular study, I only analyzed data from the question on each survey concerning the confidence students report having with regard to the triangle congruence proof. This question, more than the others, was similar in appearance and substance to the problem scored on the written exam. The close connection between these questions, I hoped, would provide some insight into students' perceived ability and comfort with proof.

Additional Teacher-Related Factors which may have Impacted Results

As a result of providing teachers with the freedom to make their own instructional decisions, classes differed in a number of ways and the results of the study should be analyzed and understood within this context. To begin with, not all teachers spent identical amounts of time instructing proof, nor were their expectations of student work necessarily equivalent. While the teacher of the four two-column classes at School B did emphasize proof in her instruction, her counterpart at School A did not. The decision not to focus on proof seems likely to have had a significant negative impact on assessment scores at School A. However, at the same time, the teacher's conscious choice to deemphasize proofs may have been informed by his experience with similar classes in previous years. Not having found much success in previous years may have encouraged him to restructure the curriculum to spend more time developing skills in which students are more likely to succeed. Another way in which the classes differed was that the two-column teacher at School B administered the common proof assessment on the day *following* the regular exam, where the ProofBlock teachers gave the assessment as part of the regular exam. While all students were aware that it would be considered part of their grade, it is possible that student attitude toward the assessment itself might have played a role in the results. As the proofs on the exam were less difficult than those at the end of that chapter in the textbook, however, it would seem that the difference of a day should not itself pose a problem for student retention.

Beginning proof by working with parallel lines rather than congruent triangles might also have affected student understanding of proof in such a way that the study did not effectively control for curriculum sequence. Each teacher used their own judgment in establishing an order for the curriculum, choosing the way that they expected would be most effective for their students. A later study investigating the issue of curriculum organization would help to provide more context for these results

As previously alluded to, the teachers involved in the study had varying levels of experience. Teachers at School B who taught with ProofBlocks were very new to the profession: one was a first year high school teacher without a strong math background, and the other was a math major and second year teacher who had taught geometry once previously and used ProofBlocks. At this school, the teacher who chose the two-column method had a math degree and three years of geometry experience. At School A, though the teacher of the two-column classes had taught for several more years than his colleague, the fourth year teacher of the ProofBlock classes had already taught Geometry A seven times. Greater levels of experience might have helped or hindered these teachers in their instruction of proof, as experience could have affected everything from classroom management and familiarity with the curriculum to expectations of students and teacher enthusiasm.

Data Analysis Methods

While the surveys asked a number of questions related to student attitude toward proof and geometry, for the purposes of this particular study I was most concerned with possible shifts in students' confidence in their ability to create correct proofs. With that in mind, I focused on the one question that asked students to rate their confidence on a proof (3C on the pre-survey and 1C on the post-survey). I tallied student responses to all the questions on the pre- and post-surveys and recorded the responses in a spreadsheet by class period. I compared the data with regard to confidence in proof by teacher and by method for each school.

The written exams were scored on a scale of 0 to 4 by the researcher and two other teachers using the rubric shown in Appendix E. The rubric was the same as that used by the Cognitive Development and Achievement in Secondary School Geometry

Project (Senk, 1985) and focused primarily on evaluating the presence of deductive reasoning. To ensure the reliability of the results, each test from one class set was scored blindly and independently by all three graders. The class set chosen for this calibration included a fairly even mixture of student proofs done in the two-column and ProofBlock formats to ensure that the graders were scoring each type in a consistent manner. Following the grading of this set of proofs, all discrepancies in scores were discussed by the group until a consensus was reached. Out of the twenty-eight tests that were graded blind, there were seven on which discrepancies in grading occurred, however, on all of these tests, all three scores were within one point of each other. Most were also related to two common student responses which the group agreed to accept as evidence of a valid deduction despite notational concerns.

The purpose of the written exam was two-fold. While its primary purpose was to allow me to determine the effect on student achievement of teaching with each method, it was also meant to serve as a check on the self-reported confidence of students in designing geometric proofs. Confidence is very subjective, and to a great extent, depends on frame of reference. Given easier quizzes and exams, one class might begin to report higher confidence than another class with equally strong skills that was being more consistently exposed to challenging material. Coupling achievement results with confidence data supplied some background for interpreting the attitudinal data.

As only one of the questions on the exam was of comparable difficulty to the proofs on the surveys (3C on the pre-survey and 1C on the post-survey), I chose to limit my analysis of the assessment data to a single proof. I used only the scores for the third exam question in my data analysis. The scores from the written exam were tallied and compared by class period, by teacher, and by method for each school.

Credibility and Validity

Having invented ProofBlocks, I had an obvious bias toward presenting ProofBlocks in the best light possible. However, I was not one of the teachers involved in teaching for the study. All of the participating teachers chose the method of instruction they planned to use prior to learning about this study. There was no extra incentive for them to teach either ProofBlocks or two-column proofs, nor was there an incentive beyond the maximization of student learning for them to raise scores on any assessment.

To ensure that students felt free to voice their opinions, all student survey responses remained anonymous. Survey data were recorded by class, not by individual student. Students put their names on the written assessments as it was to be graded by their teacher as well and recorded as part of their class grade. Assessment scores generated by the researcher and included in the survey, however, were also tallied and recorded only by class, with no student names or identifying elements attached.

CHAPTER 4: FINDINGS

The goal of this research was to explore the impact that teaching with ProofBlocks had on geometry students in two high schools. Its effects were examined with respect to student confidence and student achievement in proof. The attitudinal data came from pre- and post-surveys, and the achievement data were collected following students' completion of the chapter on triangle congruence. Due to the stark differences between the student populations at the high schools, all data are analyzed and presented for each location separately.

At School B, the number of student respondents on the assessments and the surveys remained fairly constant. Four classes were taught using each method with 125 student respondents on the assessment in ProofBlock classes and 123 respondents in two-column classes as shown in Table 2. The relatively low number of pre-survey responses in the two-column category represents a lack of pre-survey data for one class. It was beyond the scope of the study to control for a number of class-related factors including the number of students requiring special education, the number of students requiring

		Pre-Survey	**Post-Survey**	**Assessment**
School A	Two-column Respondents	22	25	33
	ProofBlock Respondents	64	68	56
School B	Two-column Respondents	87	118	125
	ProofBlock Respondents	114	129	123

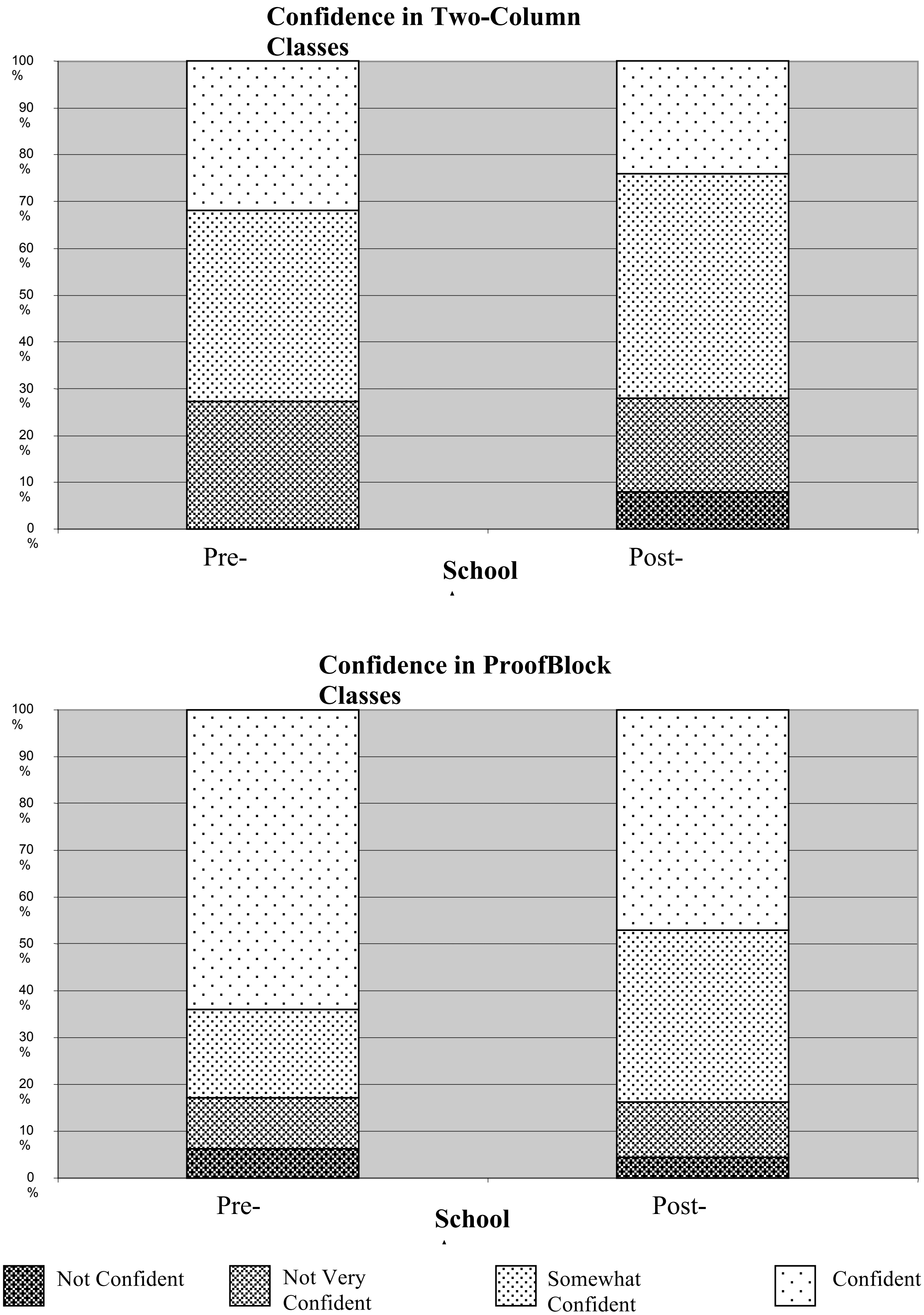
Table 2 – While the student attendance data for the pre-survey, the post-survey, and the assessment call into question the findings with regard to the assessment results at School A, the attendance at School B remained fairly consistent between the post-survey and the assessment.

English language support, and the strength of students at the beginning of the study, and data were not collected for these characteristics.

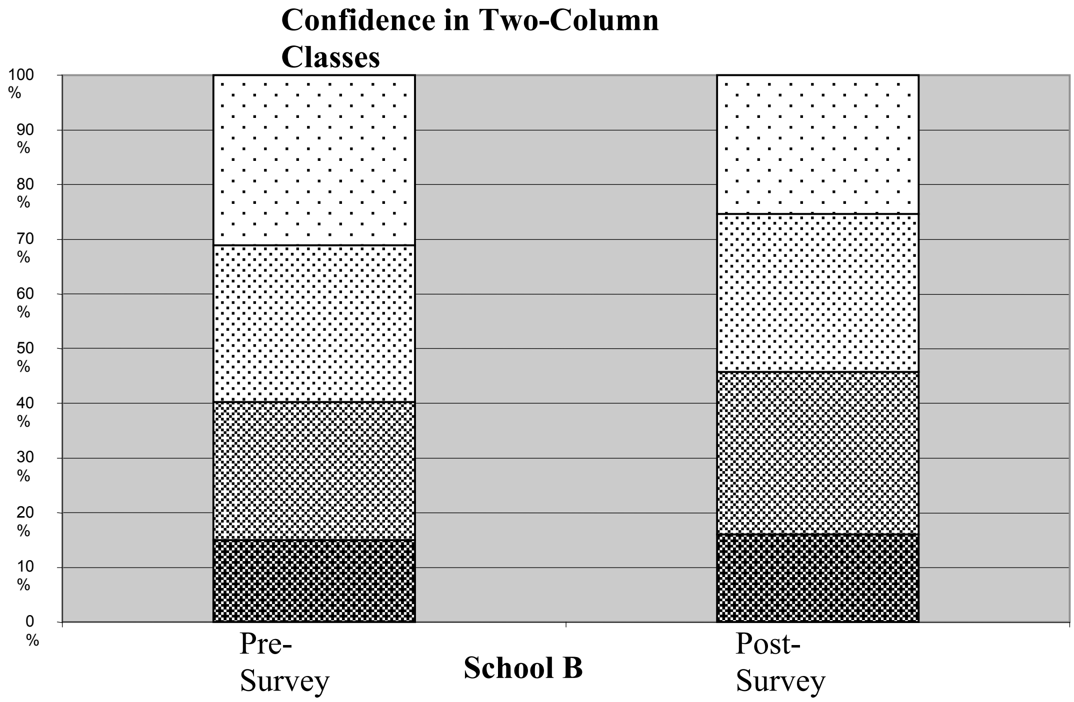
At School A, enrollment numbers were comparable between the two ProofBlock classes and one two-column class, however attendance was not steady. Only two-thirds of students taking the assessment in the two-column class were present on the day the pre-survey was administered and 12% more students were present for the post-survey in ProofBlock classes than took the assessment. Given the highly variable and comparatively low number of responding students at this location, the results of the study with regard to demonstrated confidence and achievement in proof at this site may have a high margin of error.

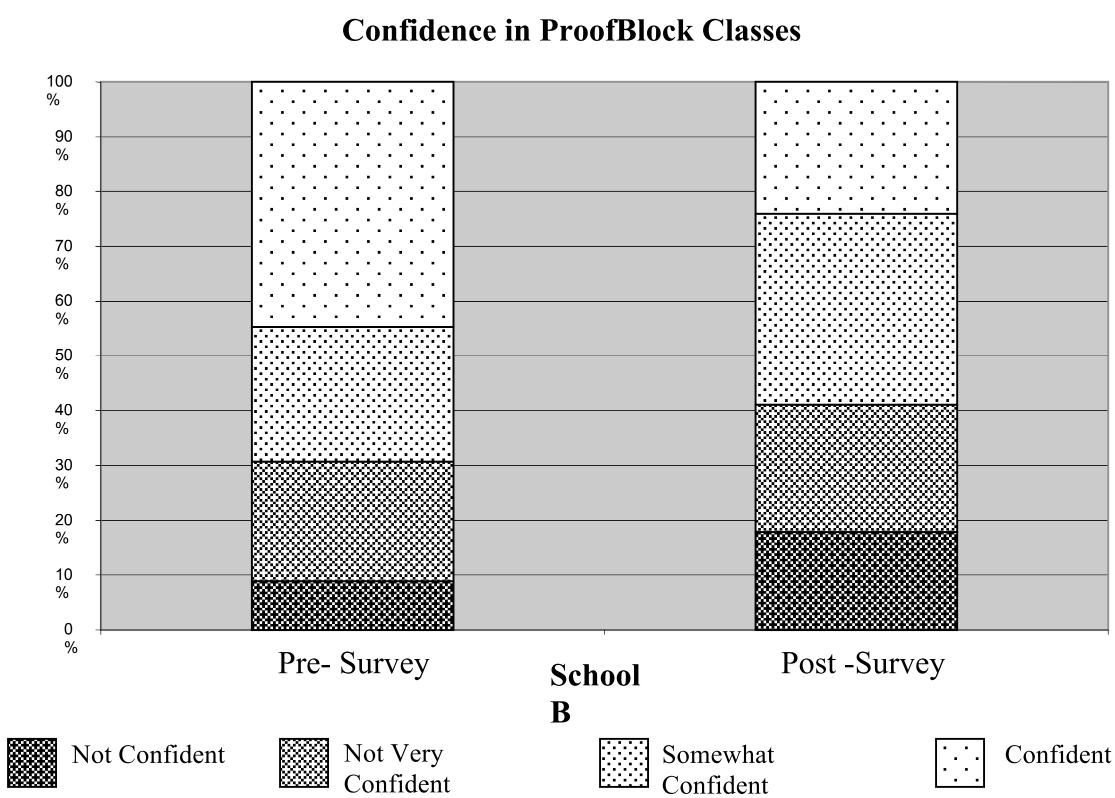
Student Confidence in Proof

For the majority of students, these geometry classes represented their first introduction to formal proof. The pre-survey data were taken solely to establish a baseline which would provide context for the post-survey results. The post-survey was administered following the unit on proving triangles congruent. It is worthy of note that students in classes using the two-column format, unlike those using ProofBlocks, had additionally completed another chapter focused on proof of angle relationships involving parallel lines and transversals, prior to the congruent triangle unit. Thus, when the post-survey and assessment data were taken, students in classes using two-column proofs had been practicing geometric proof for approximately twice as long as those in the ProofBlock classes. Pre-and post-survey data appear in Figures 5 and 6.



Figures 5a and 5b – A comparison of reported confidence on pre- and post-surveys in School A shows a decrease in confidence levels in all classes.





Figures 6a and 6b – A comparison of reported confidence on pre- and post-surveys in School B shows that in all classes, student confidence dropped, though the drop is greater in the ProofBlock classes.

Due to the anticipatory nature of the pre-survey data, it is not surprising that it differed wildly between schools, between methods, and even between classes taught by the same teacher. Students might have reported their confidence in their ability to do the pre-survey's sample proof problem (Appendix B) based on any number of factors ranging from their prior experience in the class, to what they had heard their peers say about proof. Some students may have rated their confidence in completing the proof higher because they were comparing their confidence in their ability to successfully complete a fairly simple-looking proof to their confidence that they could successfully solve the adjacent word problem, which students in both schools overwhelmingly identified as the hardest. While some students may have rated their confidence higher on the post-survey proof for this reason, I contend that defining confidence in proof with respect to the word problem was less likely to occur when students had just completed a unit full of similar problems than before students had been exposed to similar problems.

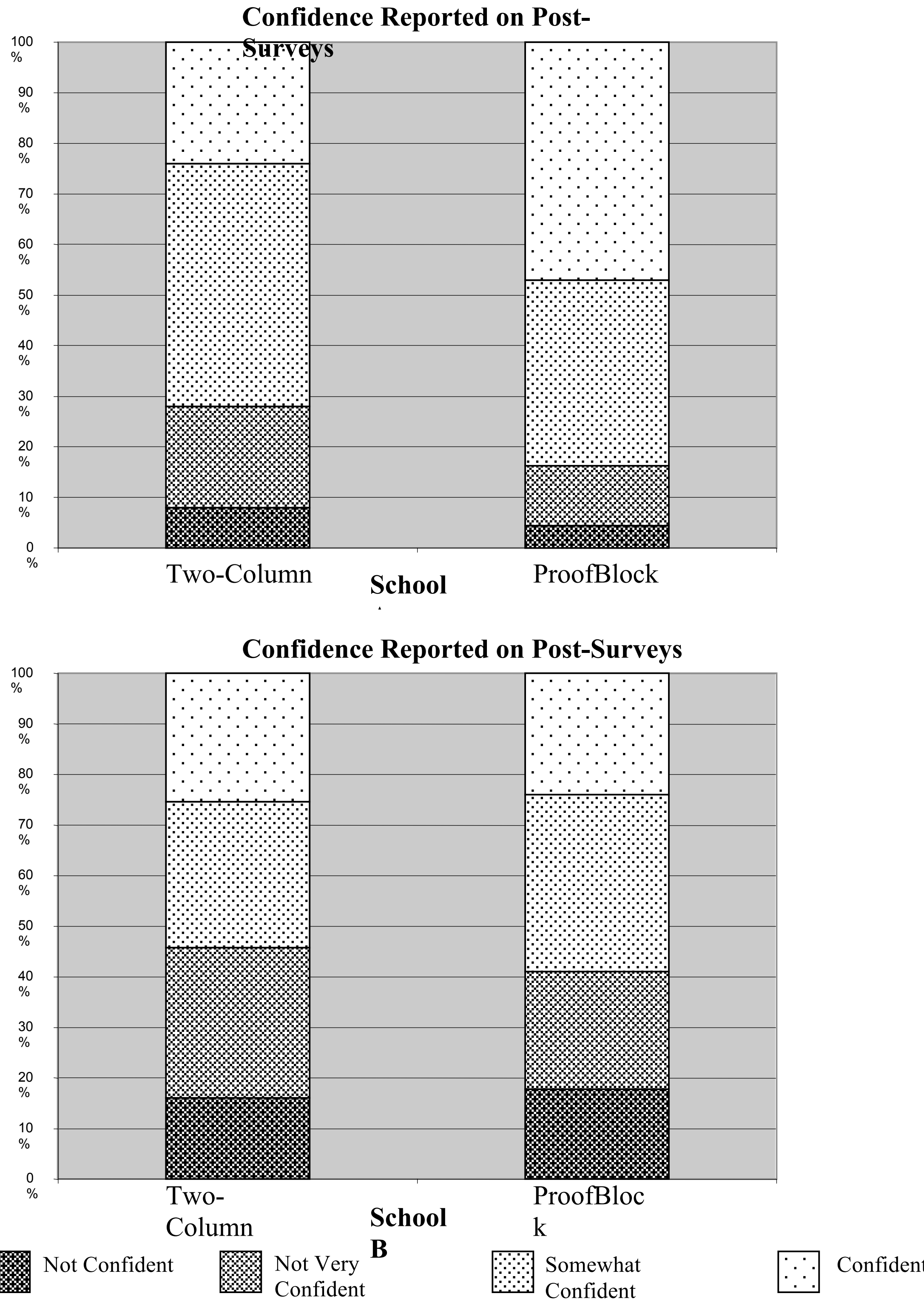
What is perhaps most worthy of note in the comparison of pre- and post-survey data is that regardless of the format of proof taught, students' initial confidence at both schools *decreased* with exposure to proof. It is beyond the scope of this study to determine whether or not this phenomenon is unique to the units relating to proof, or whether it occurs in other areas of the geometry course. However, it seems to be in keeping with prior research suggesting that nationwide most students fall short of proof mastery, many even left unable to begin a proof on their own (Senk, 1985), while at the same time, presumably greater numbers of students found success with other geometric skills.

Comparing the post-survey attitudinal data side-by-side (Figure 7), it is evident that both approaches in School B resulted in similar responses with regard to confidence. However, a similar comparison in School A yields vastly different results. In that school, over 47% of students in the two ProofBlock classes responded that they were "confident" in their ability to complete the proof on the survey as compared to 24% of students in the two-column class.

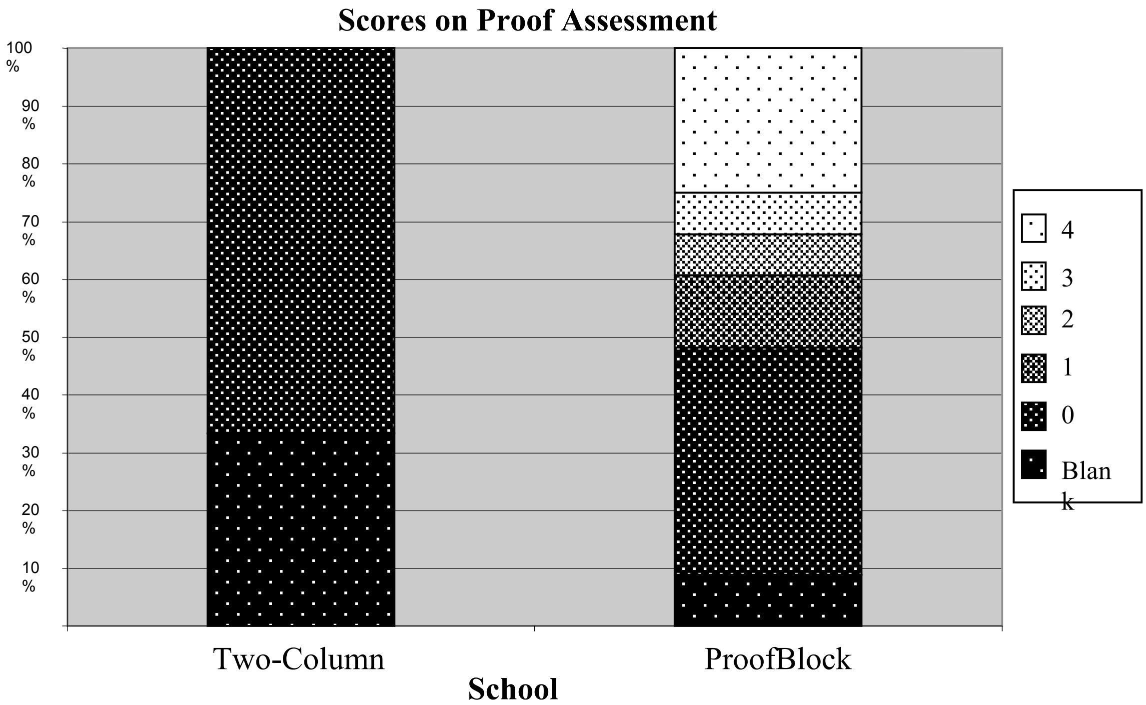
When reviewing this data, it is helpful to bear in mind that there was no real attempt at encouraging students to perform a meaningful self-assessment in the post-surveys; students were not even required to complete a proof, but only to rate their perceived ability to do so. A variety of factors might have affected these perceptions. For instance, in classes where students were consistently exposed to more difficult or complex proofs, it seems likely that some students might rate their confidence lower than others in classes where they were exposed only to basic proofs. Students perceptions might also be affected by their perceived rank in the class: those in classes with exceptionally high achieving peers might rate their confidence in proof lower because they feel less capable than their classmates. However, by asking the confidence question on the survey with regard to a specific proof, I hoped to avoid some of these issues.

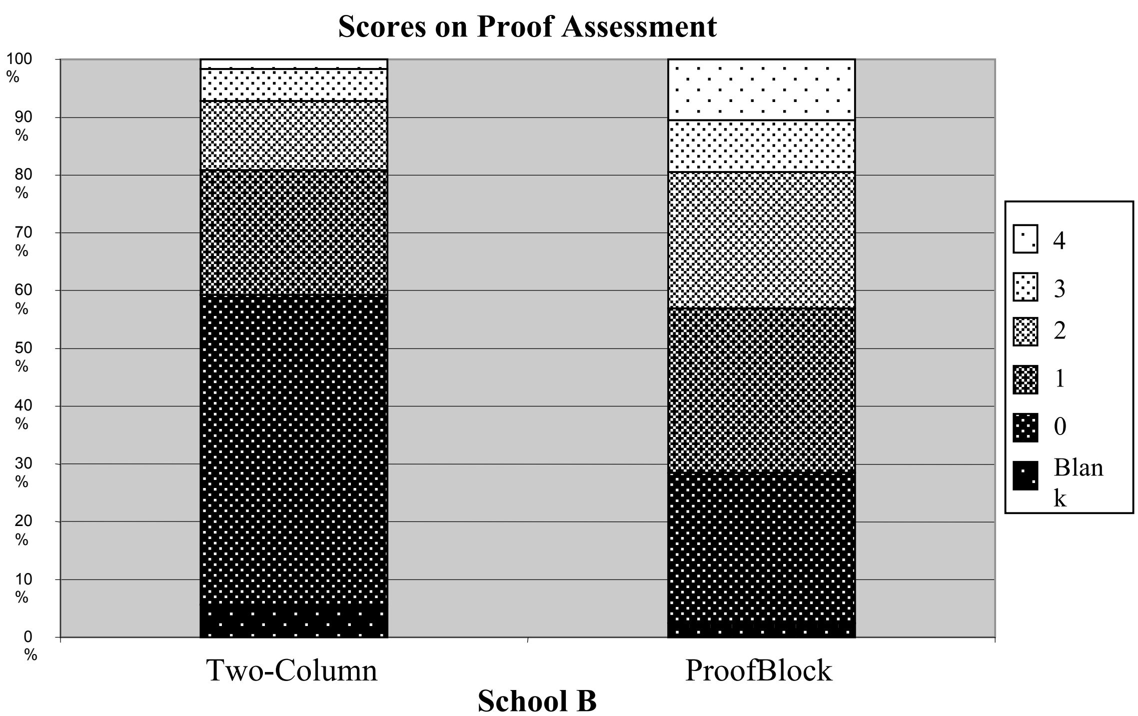
Student Achievement on the Assessment

Following the unit on congruent triangles, students in both schools took tests requiring several proofs. These assessments as well as the rubric used to score them can be found in Appendices D and E, and the relevant data are displayed in Figure 8. A score



Figures 7a and 7b – A comparison of post-survey data reveals similar confidence levels in School B classes, but a much higher level of confidence in School A's ProofBlock classes than its two-column classes.





Figures 8a and 8b – A comparison of proof assessment data reveals drastically lower scores in the two-column classes at School A, and twice as many scores of 0 or blank in two-column than in ProofBlock classes at School B.

of 3 or 4 represented a complete proof with few if any errors. A score of 2 demonstrated an ability to complete at least half the proof correctly, where papers earning a score of 1 contained only one correct, relevant, logical deduction. Prior research (Senk 1985) and my own experience suggested that many students have difficulty *starting* a chain of logical reasoning. To more fully explore this particular issue, the scorers agreed that it might be meaningful to distinguish between proofs earning a zero (meaning that they contained either entirely incorrect or correct but irrelevant deductions) and those tests which were left entirely blank.

Students in the ProofBlock classes outscored those in the two-column classes on the proof assessment by large margins at both schools. At School A, not a single student in the two-column class scored even one point on the proof, while over 30% of students in the ProofBlock classes completed the proof with a score of 3 or 4. The results were nearly as dramatic at School B, where approximately twice as many students earned 3s and 4s in the ProofBlock classes as in the two-column classes. While the scores cannot be compared between schools because the proofs graded at each site were different, the data still suggest that even when applied to classes containing vastly different student populations, introducing proof using the ProofBlock method may have some advantages over the two-column format.

CHAPTER 5: DISCUSSION

The findings of this study provide some initial promise that the use of ProofBlocks in high school classes to introduce geometric proof may result in higher levels of student achievement in proof. Students in this study were more than twice as likely to complete the proof on the final assessment if they were in a class using ProofBlocks. Additionally, the study shows that ProofBlocks did not have a negative impact on students' confidence in their ability to complete proofs.

Interpretation of Assessment Data

This study found that students at both schools who were introduced to proof using ProofBlocks were far less likely to score a 0 on the proof assessment than students who were introduced to proof through traditional instruction. One possible reason for this finding is that the ProofBlock approach provides concrete manipulatives to allow students to more easily access the abstract concepts and skills inherent in proof. The visual presentation of theorems as sets of inputs and outputs may also have helped increase student access to the process of proof by eliminating, or at least decreasing, the initial emphasis on decoding mathematical language which Moore (1994) identified as one of the key barriers to students learning proof.

Most students in the ProofBlock classes gave up the physical manipulatives relatively quickly, instead choosing to draw the shapes on paper. Eventually, as they moved from proofs of triangle congruence to the unit on parallel lines and transversals, all students transitioned to two-column proofs. In fact, many students in the classes taught with ProofBlocks actually chose to use the two-column format on their assessment. Though they moved on, it is still possible that using ProofBlocks as an introduction to geometric proof may have contributed to their later success.

While this study did not specifically examine the activities and instructional strategies employed in each classroom, it seems likely that teachers' decisions in this area impacted the results with regard to both confidence and achievement. All teachers instructing with ProofBlocks placed students in pairs and required each pair to show their work on a table-sized whiteboard for the first several days of proof. In my experience, having students work together on proofs generates mathematical discourse and encourages all students to justify their steps to each other as they work through each problem. In these ProofBlock classrooms, doing proofs was no longer an isolated activity. A second reason for ProofBlock students' success may have to do with how valid connections are spelled out specifically on each block, so that even students in the first stages of writing proofs should be able to use the manipulatives to check their work. The result is a form of instantaneous feedback, not usually available in the more traditional proof formats.

Interpretation of Confidence Data

The question of what impact teaching with ProofBlocks has on students' confidence in their proof abilities cannot be answered definitively based on the results of this study. While it appears that ProofBlocks did not negatively impact students' confidence, it is not clear that it had a consistently positive impact. In School A, levels of student-reported confidence were higher in ProofBlock classes than in the other. However, in that school, the teacher did not focus as heavily on proofs as did the others.

In School B, there was very little difference in the post-survey confidence responses between the two types of classes.

The lack of results may have been a consequence of the design of the study. Other ways of measuring student confidence might have been more accurate than the question in the survey (the limitations of which were discussed in Chapter 4). For instance, instead of just reporting their perceived confidence on a proof they had not attempted, students might instead have been asked to choose to complete one of several different problems where the problem chosen would provide information about what skills they were most confident in. Additionally, in order to form a more complete picture of students' confidence and any *shift* in confidence over the course of the units, it would be necessary to measure confidence more frequently throughout the unit. Finally, in order to use the achievement data to provide greater insight into student confidence and how the treatment affected it, the data would have to have been recorded by student instead of by class. Then, it might have been possible to look for a correlation between reported student confidence and demonstrated student achievement. Any such correlation would certainly have implications for classroom instruction.

Limitations in the Current Study and Implications for Future Research

While this study sought to examine the impact ProofBlocks had on two different populations, the small sample size, especially at one of the high schools, was problematic in determining the extent of this impact. At School A, the difficulties of having a reporting sample of only three classrooms were compounded by poor student attendance. However, with the results in the classes at this school so dramatically different with each method, it is still possible to suggest that there would likely have been a preference for ProofBlocks had all students been present.

This study compared classes within two different student populations; however it was beyond the scope of this study to compare proof performance *across* populations. As the sites were using different textbooks and had different schedules and pacing plans, each site used a test reflective of the expectations and pacing at their individual schools. Future studies seeking to use multiple sites to determine whether ProofBlocks can help bridge the achievement gap between low and high SES student populations or between racial or ethnic groups would need to administer a common assessment at all schools in order to compare student achievement between sites.

The effect which ProofBlocks has on students repeating the geometry course might also have impacted the findings of the study. This population was much larger than originally expected, especially in School A, where just over a third of pre-survey respondents fell into that category. The numbers of students repeating the course at School B hovered between four and six for each of the classes. As the numbers of repeaters were fairly evenly distributed between classes at both schools, the impact of this factor may have been felt equally in both the ProofBlock and the more traditional classes, though it was beyond the scope of this project to determine the nature or effect size of that impact.

Perhaps the greatest limitation of this study is its inability to create a complete picture of students' growth in proof understanding. While students understand proof at differing levels of complexity, this study focused on quantifying the number students who displayed at least a basic competence in proof. At the point when the post-assessment was given, all classes were no more than a third of the way through the course with regard to both time and content. The expectation was that students would continue to grow in their understanding of proof and in their ability to design creative proofs of geometric relationships as they continued their study of geometry. However, instead of tracking this growth, the study was designed only to provide brief snapshots of student performance and attitude at two points in a year-long course.

Further studies seeking to paint a more complete picture might consider a variety of factors which were beyond the scope of this project, including students' geometric understanding prior to beginning proof, level of difficulty of the proof, and content or skill retention over extended periods of time. Beyond course grades which may not accurately predict students' performance on proof, one method of gauging students' initial understanding and readiness for proof is to determine the Van Hiele level on which each student is operating. Comparing these levels upon entering and leaving the units on proof, in addition to tracking and analyzing the progress of students over time using multiple assessments, would allow researchers to explore more fully the ways in which student understanding is impacted by ProofBlocks activities. This study could also be expanded by scoring three or four proofs of varying degrees of difficulty to allow for a more nuanced understanding of students' progress, beyond just a basic level of competence. Finally, documenting students' levels of retention of the material might provide further insight into how far-reaching the possible benefits of this system could be.

Implications for Teaching

Based on these findings and given the aforementioned limitations, it seems likely that ProofBlocks results in no negative impact on student confidence and in fact holds some promise of improving student achievement in proof. While interviews were not conducted as part of this study, in a few instances, informal conversation led to some valuable teacher reflections.

One teacher who as part of the study suggested that in teaching with ProofBlocks, she had gained a more complete picture of her students' strengths and weaknesses. While in previous years she had believed students' primary difficulties were in the creation of proofs themselves, she said that teaching with this method had led her to discover gaps in students' understanding relating to the definition and application of key vocabulary concepts (the meaning of "bisector" for example). She noticed that since the students no longer had to refer endlessly back to the text to verify meanings of theorems and could instead just link the information on the blocks to check their reasoning, it became clear that her students' primary difficulty was in the process of interpreting the given information and its implication for geometric relationships in the picture. Deciding that these failings in vocabulary acquisition were the primary barrier to proof, she adjusted her instruction in the next class, and subsequently reported an increase in student achievement.

Teachers who taught with ProofBlocks also noted another phenomenon which may be worth exploring in a future study. They mentioned noticing that students seemed more self-sufficient when it came to checking their own work and assessing their own understanding, even in the early lessons on proof. Working with another student as well as being able to check their reasoning with the manipulatives are two possible explanations for this trend, if such it was. In the ProofBlock classes, the numbers of students reporting that they were confident in their ability to complete the proof on the post-survey were certainly closer to the numbers of students actually scoring 3s and 4s on the final proof. However, since the data were recorded by class and not by individual student, it is impossible to determine whether or not students reporting high levels of confidence were actually also those who scored at the highest levels on the assessment and vice versa. If future research studies conclude that this is indeed the case, it could mean a noticeable shift in teacher practice. If students are capable of checking their own work with greater accuracy, teachers will have more time to spend facilitating, assessing, and working one-on-one with students who need higher levels of support.

Although the sample size and the design of the study do not allow for findings to be generalized much beyond the population studied, the results of the research are significant insofar as they demonstrate increased achievement and no negative impact on student confidence in the classes of students introduced to proof via ProofBlocks versus a more traditional format. They provide a basis for support of a method that is both cognitively and mathematically equivalent to the more traditional formats while still providing increased access for kinesthetic learners and possibly English language learners. Additionally, the results of this study suggest that students from a variety of backgrounds can benefit from a more visual and possibly collaborative approach to proof, and that it may be worthwhile to make the effort to integrate ProofBlocks into the geometry curriculum.

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APPENDIX A

Using Blocks to Build a Proof

With ProofBlocks, each proof consists of an assemblage of blocks showing connections between information and the arguments upon which they are based. Unlike the similar flow proofs, however, the theorems and postulates are the explicit *tools* which students use in order to construct a proof step by step.

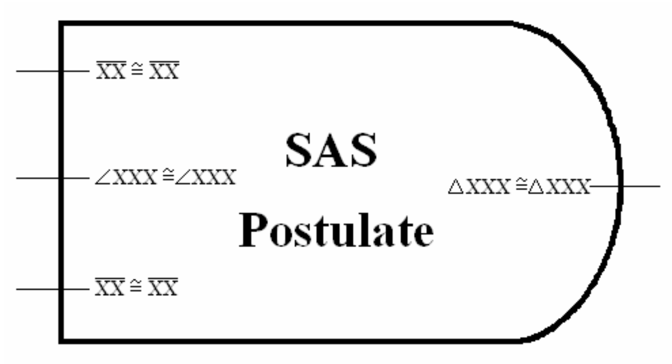
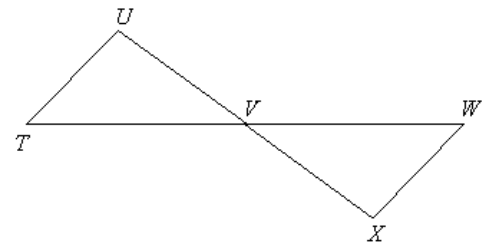


Figure 9 – The inputs and outputs of each clock are clearly labeled and whenever possible, display the general format the information will take. Once all the inputs are connected, the output has been proven true.

The heart of ProofBlocks is obviously the blocks themselves – each representing a theorem, postulate, or definition. Taken together, they act as "tool kit," which students can manipulate (physically or on paper) in order to complete a proof. Each ProofBlock has a labeled input and output that lists what kinds of information is needed in order to use it. The requirements (inputs) and the conclusions (outputs) are derived directly from the precise wording of the definition, postulate, or theorem. For example, the block for the Side-Angle-Side Postulate is shown in Figure 9. In this form, it is explicit what each theorem provides and requires with no room for misinterpretation or confusion.

With the acceptable inputs and expected outputs clearly labeled, students need only match the outputs from one block with the inputs of another to convince themselves that a logical argument has been made. The proof is complete when students reach a block with an outgoing statement that matches what they were trying to prove. To illustrate this process, consider the following example of how students might complete the very basic proof of congruent angles shown in Figure 10.



Given: $\overline{UV} \cong \overline{XV}$
$\angle UVT \cong \angle XVW$
$\overline{TV} \cong \overline{WV}$

Prove: $\angle T \cong \angle W$

Figure 10 – Most students will immediately recognize the key idea of this proof: they will need to prove these triangles are congruent.

First, the students will look at the picture, the givens, and the prove statements. The possibility of using CPCTC to find the final pair of congruent angles may or may not occur to them at this time. However, they will almost certainly suspect that the two triangles are congruent. In the tool kit of blocks before them, there are only four that have congruent triangles as an output. They separate those from the rest and then look at the given information: two sets of congruent sides and a pair of congruent angles. These givens are appropriate inputs for only one of those four blocks: SAS Postulate.

Given the three required inputs to SAS, students can see that they have completed the first part of the proof, and can now focus their attention on the second piece. The SAS Postulate proves that triangles are congruent, so they must now find a block in his toolkit that takes congruent triangles as an input. CPCTC fits that description and returns congruent segments or angles. Our students can now draw their proof as shown in Figure 11, certain that their logic is correct because all of the inputs and outputs of the blocks match. They know that they have reached the end because the final outgoing piece of information matches the prove statement in the original problem.

Reading proofs written using this technique is fairly simple with a bit of practice. A circular block with a "G" inside it indicates that the statement following it was "given" in the original problem. The information flows through the proof from left to right, each line representing a different piece of information about the problem. To use SAS, three pieces of information are needed and each enters the block on its own line as shown in Figure 11. Assuming the three congruence statements satisfy the requirements of the theorem, one logical conclusion may be reached: that the two triangles are congruent. This conclusion is shown to the right of the SAS block. Each block can also be read as an If-then statement: If $\Delta UVT \cong \Delta XVW$, then by CPCTC, $\angle T \cong \angle W$. In this way, it is trivial to convert a proof from block form to paragraph form.

Note that in the example, the student was able to start working in the middle of the proof instead of having to begin with only the givens. In my experience, when attempting proofs that involve congruent triangles, students frequently begin by choosing the congruence postulate or theorem they want to use even if the given information

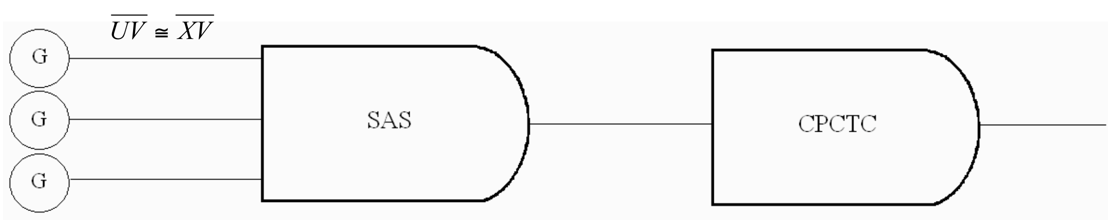


Figure 11 – Given are represented by circular blocks while SAS Postulate and CPCTC are represented by a D-shaped blocks. The information about congruent angles and segments enters the left side of the SAS block and the conclusion that the triangles are congruent appears on the right. Congruent triangles constitute the input of CPCTC which returns the conclusion that corresponding angles are congruent.

involves statements about midpoints or perpendicular bisectors. Sometimes they choose incorrectly, but when trying to connect the block they picked to the given statements, they are able to recognize and correct their error.

Process for Building Proof Blocks

Before a theorem can be used and introduced to the tool kit, it must be encoded as a proof block. While initially challenging for a class, this creation process is important because it provides opportunities for students to become familiar with formal mathematical language and get a feel for the context in which a given postulate, theorem, or definition will be useful. The first step involves identifying the hypothesis and conclusion in the formal language. Then students must anticipate the format these pieces of information will take in a problem, creating shortened or symbolic versions which will be written as inputs and outputs on the block. In addition, to choose a shape for the block, students must work beyond the theorem itself and consider whether the statement's converse is true or false.

From Words to Symbols:

To illustrate this process, consider the following example in which we will design a proof block for the definition of a midpoint. (See Figure 12 for the final block). As a first step, begin with the formal definition of a midpoint.

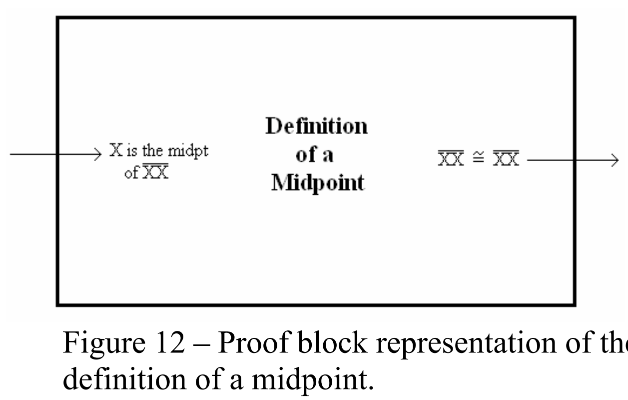


Figure 12 – Proof block representation of the definition of a midpoint.

A midpoint is a point on a line segment that divides the segment into two congruent segments.

Rewrite the definition as a conditional (or in this case, a biconditional).

A point is a midpoint of a line segment if and only if it divides the segment into two congruent segments.

The information that a point is a midpoint of a segment will be given to students in the form "__ is the midpoint of __." To assist students in identifying this statement when they see it, blanks are replaced with *X*'s equal to the number of letters that they are representing. Thus, the symbolic version written on the block will look like "X is the midpoint of $\overline{XX}$." The phrase "it divides the segment into two congruent segments" will appear in any proof as "__ ≅ __" which can be written as $\overline{XX} \cong \overline{XX}$.

Generally, the hypothesis will become the input and the conclusion will become the output of the proof block. However, since the statement is a biconditional, either piece of information could be used as the input and a valid conclusion would still follow.

A Note on Shape:

To distinguish between conditional and biconditional statements, rounded sides have been defined as outputs and straight sides typically represent inputs. When beginning a proof, information is provided in one of two ways: either it will be obvious from the diagram (the fact that two angles are a vertical angle pair, for example) or it will be specifically stated in the givens. In ProofBlocks, a circular block is used to represent both of these cases because, as the first step in a proof, a given is essentially "all output." Theorems whose converses are not necessarily true are a represented with D-shaped blocks. For instance, *corresponding parts of congruent triangles are congruent* (CPCTC) cannot be reversed to create a true statement. Thus, the straight side of the block (representing the input) must always be labeled with congruent triangles, while the congruent pieces will fall on the rounded side.

Definitions can always be written as biconditionals and, therefore, are always represented by rectangular blocks so that either condition could act as the input. Returning to our example in Figure 12, the block for the definition of a midpoint must have straight sides on the left and the right because either clause in the definition could be used as the input:

1) Given that a point is a midpoint, the two segments must be congruent, or

2) Given that the two segments are congruent, the point is a midpoint.

In practice, the rectangular manipulative is literally reversible with the same information printed on the back side of the block except for the inputs and outputs being reversed.

APPENDIX B

Students completed the pre-survey prior to beginning their first unit involving proof. The analysis described in Chapter 4 focused on student responses to question 3C.

Proof Pre-Survey

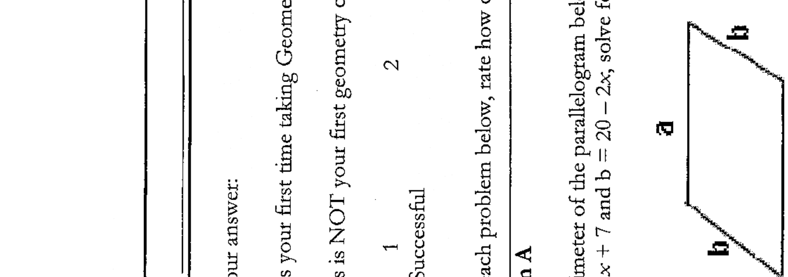
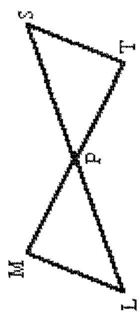
School: ______________ Your Teacher's name: ______________ Your ID #: ______________

Circle your answer:

1. Is this your first time taking Geometry? YES NO
2. If this is NOT your first geometry class, how successful have you been at doing proofs?

   1 Successful 2 3 4 Not successful

3. For each problem below, rate how confident you feel about being able to solve it. **DO NOT ACTUALLY SOLVE THEM.**

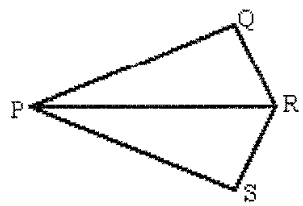
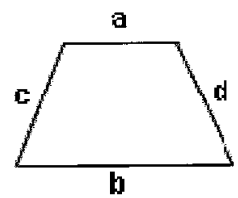
Problem A	**Problem B**	**Problem C**
The perimeter of the parallelogram below is 24. If $a = 3x + 7$ and $b = 20 - 2x$, solve for x. 	Beth Mac and her dog Trouble are exploring in the woods east of Birnam Woods Road, which runs north-south. They begin walking in a a zigzag pattern: 1km south, 1 km west, 1km south, 2 km west, 1km south, 3 km west. They walk at the rate of 4 km/hour. If they started 15 km east of Birnam Woods Road, how many minutes will it take them to get to the road?	Given: $\angle L \cong \angle S$ P is the midpoint of $\overline{LS}$  Prove that the triangles are congruent.
1 Confident 2 3 4 Not Confident	1 Confident 2 3 4 Not Confident	1 Confident 2 3 4 Not Confident

4. Which of the problems above would you MOST like to see on a test? (Circle one) A B C

   Which would you LEAST like to see? A B C

Pre-Survey (Side 1)

5. For each problem below, rate how easy or hard it looks to you.

Problem D	**Problem E**	**Problem F**
Given: $\overline{PR}$ bisects $\angle QPS$ $\overline{QP} \cong \overline{SP}$  Prove: $\angle Q \cong \angle S$	The perimeter of the trapezoid is 50. If $a = 2x - 6$, $b = 3x + 1$, $c = x + 4$, and $d = 4x - 12$, Solve for x. 	Jiminey Cricket is caught in a windstorm 500 cm from his home. Each time he jumps toward home he leaps a distance of 50cm, but before he regains strength to jump again, he is blown back 40 cm. If it takes a full minute between jumps, how long will it take Jiminey to get home?
1 Easy 2 3 4 Hard	1 Easy 2 3 4 Hard	1 Easy 2 3 4 Hard

6. Based on your knowledge of the class at this point, how confident do you feel about this subject?

1 2 3 4 5 6 7 8

Confident ... Not Confident

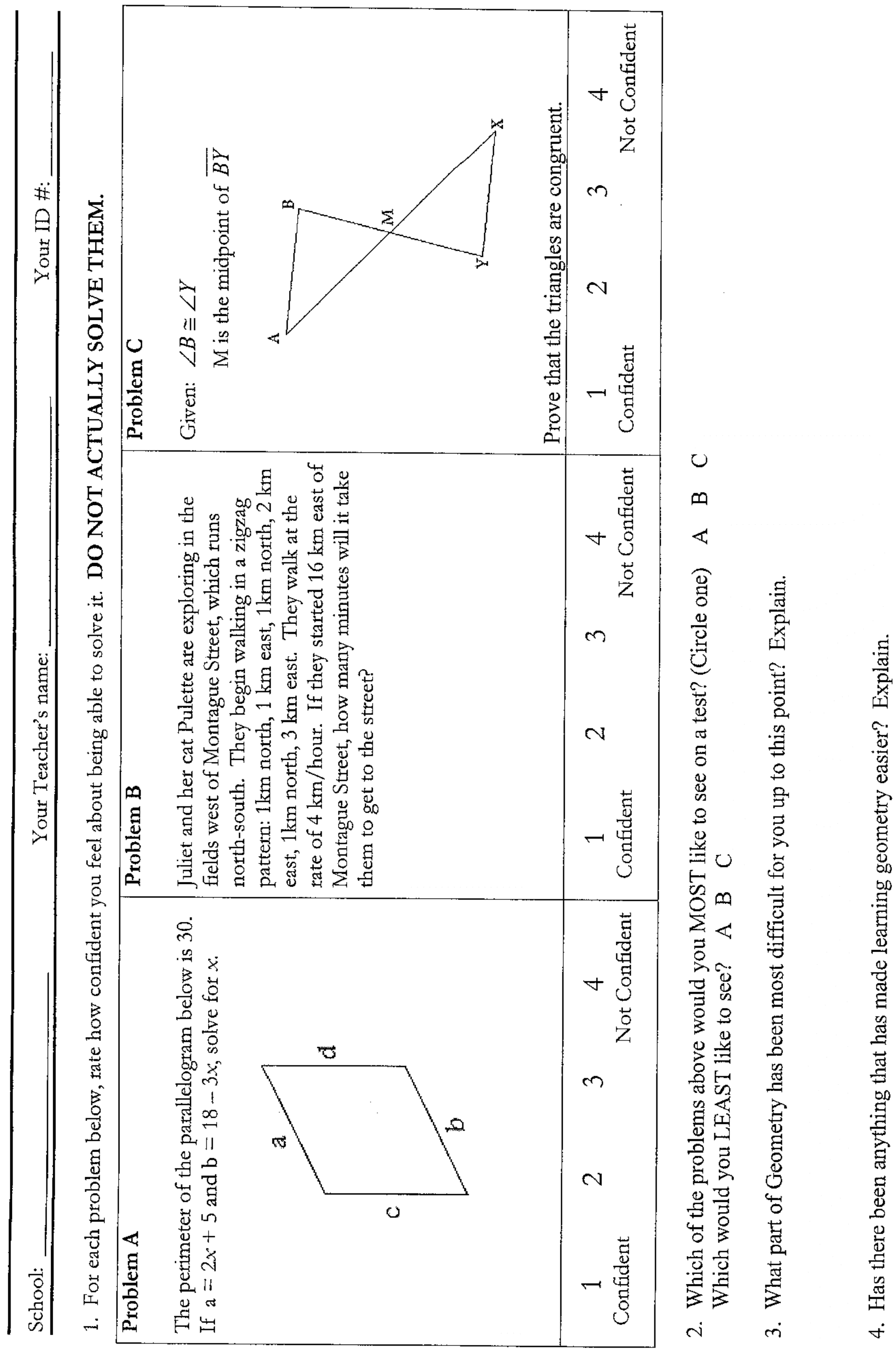
7. Given a choice between taking Geometry and taking any other math, please rank your preference for taking Geometry.

1 High Preference for Geometry   2   3   4 Low Preference for Geometry

Pre-Survey (Side 2)

APPENDIX C

Students took the post-survey upon completion of the unit on triangle congruence. The analysis in Chapter 4 focuses on students' responses to question 1C.



Post - Survey

School: ______ Your Teacher's name: ______ Your ID #: ______

1. For each problem below, rate how confident you feel about being able to solve it. **DO NOT ACTUALLY SOLVE THEM.**

Problem A	Problem B	Problem C
The perimeter of the parallelogram below is 30. If $a = 2x + 5$ and $b = 18 - 3x$, solve for x.	Juliet and her cat Pulette are exploring in the fields west of Montague Street, which runs north-south. They begin walking in a zigzag pattern: 1km north, 1 km east, 1km north, 2 km east, 1km north, 3 km east. They walk at the rate of 4 km/hour. If they started 16 km east of Montague Street, how many minutes will it take them to get to the street?	Given: $\angle B \cong \angle Y$; M is the midpoint of $\overline{BY}$. Prove that the triangles are congruent.
1 Confident 2 3 4 Not Confident	1 Confident 2 3 4 Not Confident	1 Confident 2 3 4 Not Confident

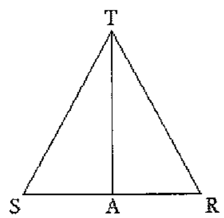
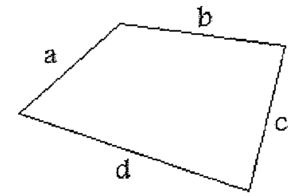
2. Which of the problems above would you MOST like to see on a test? (Circle one) A B C
   Which would you LEAST like to see? A B C

3. What part of Geometry has been most difficult for you up to this point? Explain.

4. Has there been anything that has made learning geometry easier? Explain.

Post-Survey (Side 1)

5. For each problem below, rate how easy or hard it looks to you.

Problem D	Problem E	Problem F
Given: $\overline{AT}$ bisects $\angle STR$ $\overline{TS} \cong \overline{TR}$  Prove: $\angle S \cong \angle R$	The perimeter of the quadrilateral is 40. If a = $3x - 5$, b = $2x + 1$, c = $x + 8$, and d = $2x - 9$, Solve for x. 	Hoppy the Frog is stuck at the bottom of a well 26 feet below ground. Each time he jumps toward the surface he leaps a distance of 3 feet, but the well is slippery and he slides back 1 foot. If it takes a full minute between jumps, how long will it take Hoppy to get free?
1 (Easy) 2 3 4 (Hard)	1 (Easy) 2 3 4 (Hard)	1 (Easy) 2 3 4 (Hard)

6. Based on your knowledge of the class at this point, how confident do you feel about this subject?

1 2 3 4 5 6 7 8

Confident ... Not Confident

7. Given a choice between taking Geometry and taking any other math, please rank your preference for taking Geometry.

1 High Preference for Geometry 2 3 4 Low Preference for Geometry

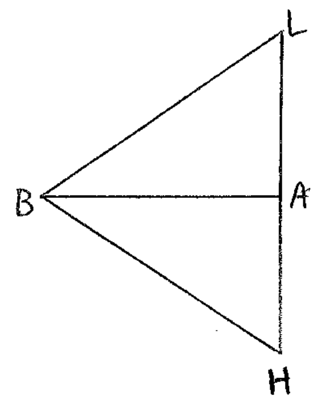
Post-Survey (Side 2)

APPENDIX D

Students at both high schools were given a written assessment following the unit on proving triangles congruent. All students at each site took the same assessment. One problem from each assessment was scored using the rubric in Appendix D. The assessment at School A had only two problems while the assessment at School B had four problems. What follows is a blank version of each test and then a sample of a test completed in the ProofBlock format.

Given: $\overline{BA}$ is the perpendicular bisector of $\overline{LH}$

Prove: $\triangle BLA \cong \triangle BHA$

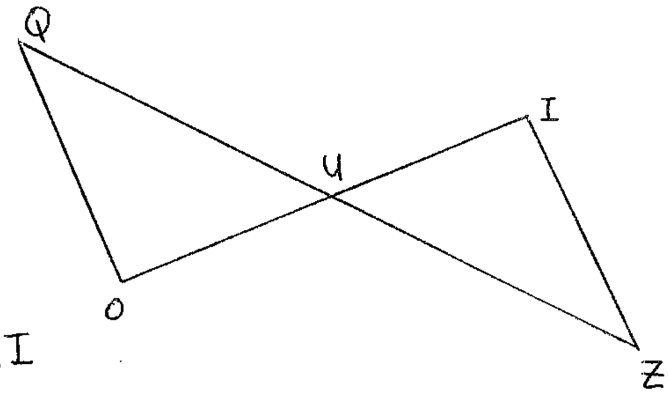


Two-Column

School A Assessment (Side 1)

Given: U is the midpoint of $\overline{QZ}$; $\angle Q \cong \angle Z$

Prove: $\triangle QUO \cong \triangle ZUI$



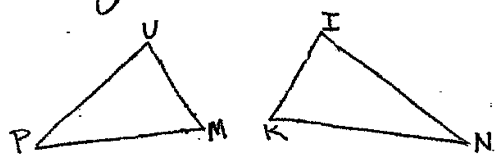
Two Column

School A Assessment (Side 2)

CONGRUENT △ TEST

label diagrams with congruence information.

1)



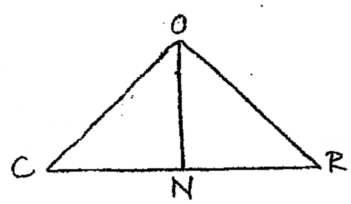
Given: $\overline{PU} \cong \overline{IN}$

$\angle U \cong \angle I$

$\angle M \cong \angle K$

Prove: $\triangle PUM \cong \triangle NIK$

2)



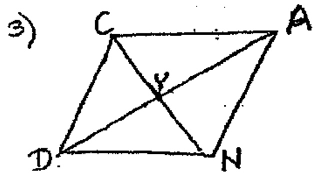
Given: $\overline{ON}$ bisects $\angle COR$

$\overline{CO} \cong \overline{OR}$

Prove: $\triangle CON \cong \triangle RON$

School B Assessment (Side 1)

3)

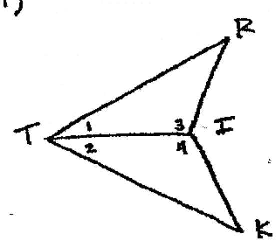


Given: $\overline{CN} \perp \overline{DA}$

Y is the midpoint of $\overline{AD}$

Prove: $\triangle CAY \cong \triangle CDY$

4)



Given: $\angle 1 \cong \angle 2$

$\angle 3 \cong \angle 4$

Prove: $\overline{RI} \cong \overline{KI}$

(5) What does CPCTC mean?

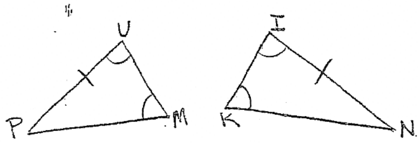
School B Assessment (Side 2)

CONGRUENT △ TEST

Label diagrams with the congruence information

55/55 GOOD JOB

1)



Given: $\overline{PU} \cong \overline{IN}$

$\angle U \cong \angle I$

$\angle M \cong \angle K$

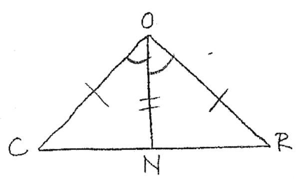
Prove: $\triangle PUM \cong \triangle NIK$

(G) $\angle M \cong \angle K$

(G) $\angle U \cong \angle I$ — AAS — $\triangle PUM \cong \triangle NIK$

(G) $\overline{PU} \cong \overline{IN}$

2)



Given: $\overline{ON}$ bisects $\angle COR$

$\overline{CO} \cong \overline{OR}$

Prove: $\triangle CON \cong \triangle RON$

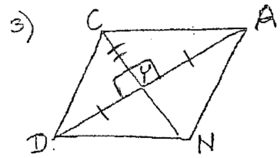
(G) $\overline{CO} \cong \overline{OR}$

(G) $\overline{ON}$ bis. $\angle COR$ $\boxed{\text{def of bis.}}$ $\angle CON \cong \angle RON$ — SAS — $\triangle CON \cong \triangle RON$

(P) $\boxed{\text{refl.}}$ $\overline{ON} \cong \overline{ON}$

School B Assessment – Sample Student Response (Side 1)

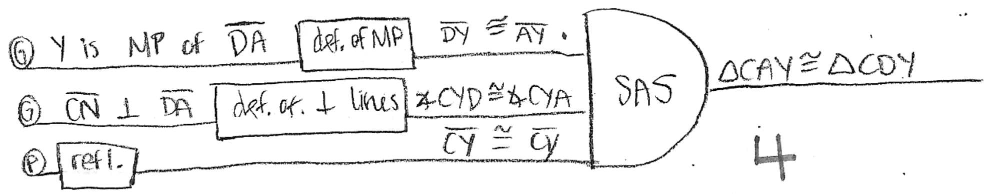
3)



Given: $\overline{CN} \perp \overline{DA}$

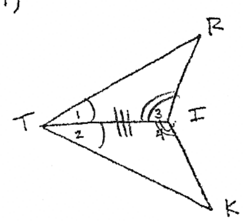
Y is the midpoint of $\overline{AD}$

Prove: $\triangle CAY \cong \triangle CDY$



4

4)

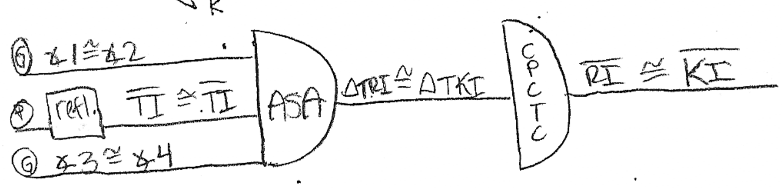


Given: $\angle 1 \cong \angle 2$

$\angle 3 \cong \angle 4$

Prove: $\overline{RI} \cong \overline{KI}$

4



5) What does CPCTC mean?

corresponding parts of congruent triangles are congruent

School B Assessment – Sample Student Response (Side 2)

APPENDIX E

Rubric for grading written proof assessment in Appendix C.

0 — Student writes nothing, writes only the “given,” or writes only invalid or useless deductions.

1 — Student writes at least one valid deduction and gives reason.

2 — Student shows evidence of using a chain of reasoning, either by deducing about half the proof and stopping or by writing a sequence of statements that is invalid because it is based on faulty reasoning early in the steps.

3 — Student writes a proof in which all steps follow logically but in which errors occur in notation, vocabulary, or names of theorems.

4 — Student writes a valid proof with at most one error in notation.

(Senk, 1985)